# The Stellar MRI

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Despite its importance for the evolution and final end states of stars, the physics of stellar angular momentum transport remains poorly understood. Recent advances in ensemble asteroseismology has yielded measurements of core rotation in many ascending red giant branch stars (RGB) Mosser et al. (2012). The main result (Fig 1) is that the cores of these stars are rotating at least 10 times slower than what is predicted by state-of-the art models (Cantiello, Mankovich, Bildsten, Christensen-Dalsgaard, & Paxton, 2014). Moreover, detailed asteroseismic modeling shows that the bulk of radial differential rotation is localized in the radiative region between the H-burning shell and the bottom of the convective envelope Klion and Quataert (2016).

The magnetorotational instability (MRI) has been extensively studied in the context of angular momentum transport in accretion disks. However, it is not clear what role–if any–it might play in the radiative regions of differentially rotating stars. The existing literature is limited Kagan and Wheeler (2014); Wheeler, Kagan, and Chatzopoulos (2015), and attempts to simulate this mechanism in stellar interiors have not yielded complete results appropriate for inclusion in stellar evolution codes.

We propose to investigate the role of the MRI in stars using simulations that account for the important physical ingredients of stellar radiative regions: stable radial stratification, mean molecular weight gradients, differential rotation, and magnetic fields. These effects are parameterized by the Brunt-Väisälä frequency $N$, background gradient $\nabla_\mu$, shear parameter $q = d\log\Omega/d\log R$, and plasma $\beta$. Menou, Balbus, and Spruit (2004) emphasized the importance *double-diffusive* effects in the stellar MRI. These effects occur when there is an imbalance between the microphysical dissipation of momentum $\nu$, magnetic field $\eta$, and heat $\chi$. In their analysis, they note that double diffusive effects can both stabilize and destabilize depending on the relative ratio of these coefficients. Here, we will carefully consider the effects of double-diffusive effects by a series of fast, linear solutions to characterize the parameter space most relevant to stellar parameters.

Using the Dedalus framework, we will simulate the MRI in a simplified geometry to extract effective torques over a range of parameters. We will then implement these results in a 1D implementation in the MESA code to test against the full set of observations available: the core rotation of red giants, the solar core rotation profile, and the final spin rate of compact remnants.

This project is extremely ambitious owing to the many length scales that must be resolved simultaneously and the very large parameter space to be spanned. However, we have at our disposal the ideal tools to make significant progress: the spectral magnetohydrodynamic simulation framework Dedalus and the extremely flexible stellar evolution code MESA. Dedalus provides an excellent platform to study the MRI in stellar interiors: it can be adapted to include the new terms and its spectral accuracy ensures that the widest possible dynamic range for a given resolution (Fig. 2). Thanks to the extraordinary results from Kepler asteroseismology and the versatility of the open stellar evolution code MESA, we will be able to immediately test our results against the observations. This project could lead to a novel theory for internal angular momentum transport in stellar interiors, and, among other results, to updated predictions for SN and GRB progenitors.

Both the Dedalus and MESA codes are fully open, so our results will be fully testable and reproducible by the astrophysics community. We will publish not only our results, but also our input files and configuration for both codes.

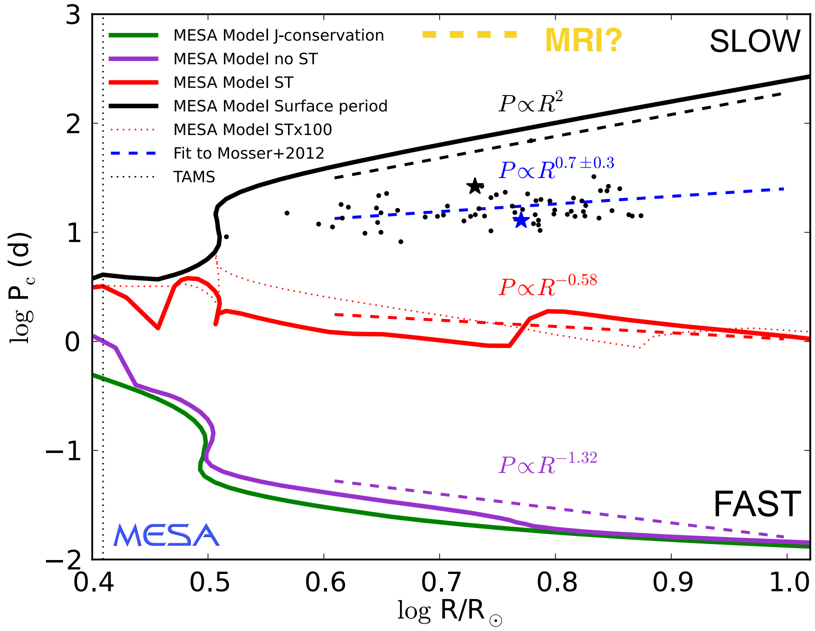


Figure 1: MESA calculations of the evolution of the average core rotational period as a function of stellar radius for different assumptions of angular momentum transport in a $1.5\,M_\odot$ model initially rotating at $50\,\mathrm{km\,s^{-1}}$. None of the current theories implemented in stellar evolution calculations can reproduce the observations of Mosser et al. (2012) (black dots).

# 1 Numerical Simulations of MRI

The MRI was first considered in the stellar context by Balbus and Hawley (1994). Menou et al. (2004) extended the analysis to consider the effects of viscous, magnetic, and thermal diffusion. More recently, simulations by Obergaulinger, Aloy, Dimmelmeier, and Müller (2006); Obergaulinger, Cerdá-Durán, Müller, and Aloy (2009) and analytic work by Wheeler et al. (2015) have been applied to the study of massive stars, while Spada, Gellert, Arlt, and Deheuvels (2016) used the results of a single simulation of the MRI to calibrate a prescription for internal angular momentum transport. The latter group showed encouraging results when compared to the observed red giant cores rotation rate. However, this stands on MRI simulation that do not include the important effects of mean molecular weight, particularly relevant in the differentially rotating regions of ascending RGB (e.g. Cantiello et al., 2014).

## 1.1 Angular Momentum Transport: Homogeneous Case

We will begin by evolving the MRI in the case of a chemically homogeneous, radially stratified Taylor-Couette geometry. Building on the previous experience of proposer Oishi in using Dedalus to study the MRI Clark and Oishi (2016a, 2016b), we will perform a suite of simulations aimed at reproducing growth rates previously calculated by Menou et al. (2004).

### 1.2 Angular Momentum Transport: Mean Molecular Gradient

Next, we will produce a series of simulations of increasing background mean molecular weight. We will quantify how this additional physics stabilizes (or doesn't) the MRI under conditions relevant to RGB stars. Using our findings, we will construct a parameterization of the turbulent viscosity $\nu_T$. This will allow us to characterize the torque from MRI driven turbulence in stellar interiors.

### 1.3 Chemical Mixing

Along with our MHD equations, we will consider the diffusion of a passive scalar. We will add this after the MRI reaches saturation in order to understand the nature of mixing induced by the instability. We will produce a measurement of the Schmidt number $\mathrm{Sc} = \nu_T/D_T$, where $\nu_T$ is the turbulent viscosity and $D_T$ is the turbulent diffusion of mass. This will allow to further constrain the MRI mechanism, for example by comparing with the observed surface nitrogen enrichment in main sequence massive stars (Hunter et al., 2008).

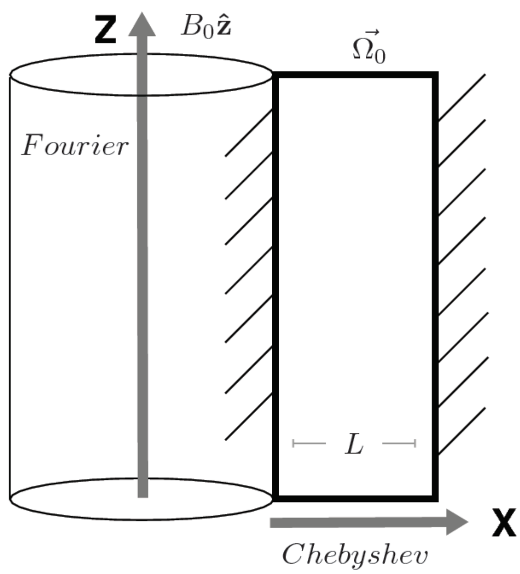


Figure 2: Schematic representation of the computational setup. The Dedalus code uses Chebyshev polynomials in the radial direction, and Fourier polynomials in the azimuthal and axial domains.

## 2 MESA Implementation of MRI and Observational Tests

Proposer Cantiello will implement a diffusion approximation representation of the MRI results in the 1D code MESA. This will permit to test angular momentum transport from MRI against the observations of RGB core rotation rates. If the implementation will pass this test, we will also run calculations to check if the MRI can reproduce the flat rotation rate of the solar radiative core, the final spin of WDs and young NSs (Suijs et al., 2008) and is consistent with the surface chemical enrichment of rotating massive stars (Brott et al., 2011).

## 3 Potential Applications

This project could lead to the solution to a long-standing problem in stellar astrophysics. A tested, novel implementation for internal angular momentum transport is required to make quantitative predictions about the state of the stellar core before core collapse. The chemically homogeneous channel for the formation of BH-BH binary mergers relies on the amount of internal mixing and angular momentum transport in stellar interiors, so our work may also improve our quantitative understanding of potential aLIGO sources.

## 4 Personnel

Oishi will run the MHD simulations and produce the torque prescription. He will use a series of 1D and 2D simulations to identify areas of parameter space most relevant to post main-sequence stellar interiors, including $\nabla_\mu$. Once appropriate parameters are identified, he will run full 3D MHD simulations to construct torque and mixing models from $\nu_T$ and Sc respectively. Oishi is a founding member of the Dedalus collaboration and has significant experience with MHD turbulence in general and the MRI in particular.

Cantiello will implement in MESA a 1D diffusion approximation of the MRI angular momentum transport and chemical mixing results. He will then run stellar evolution models and test the results against, among others, asteroseismic observations. Cantiello is an expert in stellar evolution, asteroseismology, and a member of the MESA council.

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