A Derivation of the Hubble- Lemai tre Law that Identifies Dark Energy and Dark Matter

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Abstract

I apply a generalized Friedman-Lemai tre-Robertson-Walker (FLRW) equation of the line element to obtain a radially symmetric form of a gravitationally perturbed Robertson-Walker equation of the linear element that accommodates the Hubble expansion. We show that, generally within this space-time, the gravitational field has an equation of state (EoS) of $w = -1$. This motivates the exploration of the suitability of the gravitational field to assume the role of dark energy. Here, the unique coupling of the gravitational field with space-time, by means of the metric

$g_{\mu\nu}$is considered a favourable factor of great significance. Of great utility in this work is the positive feedback gravitational mechanism invoked in Einstein's seminal work - *The Foundation of the Theory of General Relativity.* This mechanism gives rise to the induction by, and of, gravity: gravitational auto-induction. Auto-induction is a process of gravitation induced by the gravitational field - that was itself initially induced by a material body - whereby, this gravitational field acts just like baryonic matter and radiation do in their induction of gravitational fields. The mechanism of auto-induction leads naturally to the continuous growth of the gravitational field, even in the absence of changes in the gravitating body. This auto-induced growth of the gravitational field is here proposed to manifest itself in the empirically measured phenomena of the accelerating expansion of the universe and of the unexpected velocity profiles of orbits of bodies within many galaxies . Both of these phenomena are here explained by the auto-induced growth of the gravitational field, under the constraint of the conservation of energy density, and so into emergent spaces. The mechanism of this expansion of the space of the observable universe is the creation, by gravitational pressure, of new spaces everywhere. The expansion, by virtue of the laws of conservation of gravitational energy density and the *EoS* of gravity, is shown to be locally isobaric. The Hubble-Lemai tre law is derived from the locally isobaric nature of the Hubble expansion. This leads to the discovery that the Hubble parameter is invariant and, as such, is a cosmic Hubble constant. The Hubble constant references a universal maximum magnitude, both of the gravitational pressure and of the gravitational energy density, expressed as $H^2/2k$. Bodies and fields of all scales of mass-energy and energy density participate in this maximum gravitational energy density. This maximum energy density of gravity is here shown to be approached in the large empty spaces that dominate the observable universe. In such regions, the universal metric here applied recovers the metric of the *de* Sitter space-time of an empty expanding universe. The expansion of the gravitational field, and so of space, over time lends a historical character to the laws of gravitation and so also to the expressions of the descriptions of the kinematics of orbiting bodies. I derive the historical forms of certain of these laws of nature. Continuous expansion of the field and so of its form - space - is essential to preventing their collapse.

1.0 INTRODUCTION

There is no widely accepted single theory that explains the sets of gravitational phenomena separately attributed to dark energy and dark matter. So, there occurs a dichotomy within gravitational science. At large scales, the expanding space-time of FLRW theory, without spatial curvature, through its descriptions of spatial expansion, continues to be affirmed by observations. However, at small (galactic) scales, it is not applicable.

Theoretical explorations of the relationships of space-times applicable to galaxies, on one hand, and, on the other, to the FLRW space-time have not produced very useful results. In fact, in 1945, Einstein and Straus established the conditions for the coexistence of the spherically symmetric curvilinear static Schwarzschild space-time - that explained significant phenomena at the scale of our solar system - and the spatially flat FLRW space-time [1]. However, the two had to be mutually excluded in space and they only had external relations with the small-scale Schwarzschild space-time simply existing within the FLRW space-time. It has been shown that such coexistence was very fragile for the Schwarschild space-time, with instability under isotropic radial changes and vulnerability to non-spherical perturbations [2]. So, these space-times had to be separated by having the small-scale one existing in a spherical *vacuole* within the cosmic FLRW space-time [3].

In the absence of a robust coherent connection between the space-times of large and small scale regions, there has been a reassertion, in the latter, of Newtonian science of gravity in cosmology and astrophysics. For, at small scales, it is apparent that Newtonian science of gravity is the normative model applied in analysis and so in simulations. So, though historically based on Einstein's theory of gravity, modern cosmology was, in small-scale regions, mainly applying Newton's methods against the background of a flat space-time. This occurs even though Newtonian physics had already been proven incapable of explaining certain subtle gravitational effects such as the precession of Mercury's orbit and the deflection of light around massive bodies that were explained by general relativity (GR) in [4].

Yet, however contradictory the situation appears, it is quite comprehensible. For, in GR, orbits require inherently curvilinear space-times [4]. So, in GR, orbits are inexplicable in the currently applicable FLRW space-time (that is spatially flat) since the latter may, simply by means of a time coordinate transformation, be shown to become, thereby, a Minkowskian flat space-time. So, GR cannot be applied to explain galactic orbits in the GR-derived FLRW theory of a spatially flat universe. However, since Newton's science of gravity applies only to flat space and, moreover, is the only alternative to Einstein's, then its reassertion is fundamentally unavoidable in FLRW cosmology. This situation amounts to an internal theoretical incongruity in modern cosmology and astrophysics.

It is here proposed that this fundamental incongruity within cosmology and astrophysics is the primary cause of the persistence of the mysteries of dark energy and dark matter.

So, I seek to develop a unitary explanation within the paradigm of the gravitational theory of GR that explains both sets of phenomena. The desire is to include only the universal empirical element of the

Hubble expansion of space [5] within a strictly classical GR explanation that treats all matter generically as energy tensors, while generally ignoring the other aspects of the different species of matter, some of which become so important in FLRW cosmology.

Furthermore, it was recognized that in developing his general field equations, Einstein had invoked a feedback mechanism in the induction of gravity [4]. It seemed that, given the tight coupling of gravity with space by means of the metric, such a mechanism had the potential to explain spatial expansion.

It was clear that the assumed homogeneity in FLRW cosmology is only approximated at very large scales in the universe. So, in order to accommodate small-scale - that is, orbital - phenomena [6], the principle of homogeneity had to be surrendered.

However, the universe still appears fairly isotropic as observed from within our solar system and since galaxies display galactocentric kinematics and distributions of matter, then a radially symmetric metric seemed appropriate. Most important, however, is the fact that the cosmic expansion is isotropic.

The rigidity of an absolutely flat spatial manifold was abandoned for clearly that does not apply within galaxies with their strongly curvilinear geodesics.

Finally, the large-scale curvature of space is not observationally supported and was not pursued here.

So, it was that I chose to combine a generalized form of FLRW metric with a scale factor, but without a curvature term. The generalization was intended to relax the constraints applied in the FLRW metric so as to widen the scope of candidate space-times for theoretical exploration.

2.0 A GENERALIZATION OF THE FLRW EQUATION OF THE LINEAR ELEMENT

In satisfying the empirically supported universal isotropy, consider a universe with a radially symmetric cosmic linear element of the form:

$$(ds)^2 = g_{tt}(dx^0)^2 + g_{rr}[(dx^1)^2 + (dx^2)^2 + (dx^3)^2] \tag{1}$$

$$= g_{tt}(dx^t)^2 + g_{rr}(dx^r)^2 \tag{2}$$

Where:

$$dx^r = [(dx^1)^2 + (dx^2)^2 + (dx^3)^2]^{1/2} \tag{3}$$

The line element - ds - is the invariant proper distance, or duration, corresponding to the infinitesimal coordinate vector components dx^ν by means of the $g_{\mu\nu}$ that are components of the metric, with tensorial indices $\mu, \nu = 0, 1, 2, 3$. Equation (1) generalizes the curvature-free generic FLRW equation of the linear element that has $g_{tt} = 1$ and $g_{rr} = -a^2$, where $a = a(t)$ is the scale factor.

(The italicized letter t is used to variously represent coordinate time in seconds and the gravitational energy pseudo-tensor, as well as to index time components of tensors, pseudo-tensors, and of the gravitational field. The context should provide the necessary indication.)

3.0 FIELD AND ENERGY COMPONENTS GRAVITY: EQUATION OF STATE

The field components of gravity are defined as:

$$\Gamma^{\tau}_{\mu\nu} = -\frac{g^{\sigma\tau}}{2}\left(\frac{\partial g_{\mu\sigma}}{\partial x^{\nu}} + \frac{\partial g_{\nu\sigma}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}}\right) \tag{4}$$

(I use the original definition of the gravitational field component given in [4]. It is the negative of the formula given by [7].)

With substitutions of the tensorial indices - $0, 1, 2$, and 3 - by those of the coordinate system - r and t, we first get:

$$\Gamma^{r}_{rr} = -\frac{g^{rr}}{2}\left(\frac{\partial g_{rr}}{\partial x^{r}} + \frac{\partial g^{rr}}{\partial x^{r}} - \frac{\partial g_{rr}}{\partial x^{r}}\right) = -\frac{g^{rr}}{2}\frac{\partial g_{rr}}{\partial x^{r}} \tag{5}$$

Similarly obtained, the other field components are:

$$\Gamma^{t}_{tt} = -\frac{g^{tt}}{2}\frac{\partial g_{tt}}{\partial x^{t}} \tag{6}$$

$$\Gamma^{r}_{tr} = \Gamma^{r}_{rt} = -\frac{g^{rr}}{2}\frac{\partial g_{rr}}{\partial x^{t}} \tag{7}$$

$$\Gamma^{t}_{rt} = \Gamma^{t}_{tr} = -\frac{g^{tt}}{2}\frac{\partial g_{tt}}{\partial x^{r}} \tag{8}$$

$$\Gamma^{r}_{tt} = \frac{g^{rr}}{2}\frac{\partial g_{tt}}{\partial x^{r}} \tag{9}$$

$$\Gamma^{t}_{rr} = \frac{g^{tt}}{2}\frac{\partial g_{rr}}{\partial x^{t}} \tag{10}$$

Within the coordinate constraint of $\sqrt{(-g)} = 1$, (that will generally hold going forward), where g is the determinant of the metric, the energetic flux densities of gravity are given in [4] as:

$$t^{\alpha}_{\sigma} = \frac{1}{\kappa}\left(\frac{\delta^{\alpha}_{\sigma}}{2} g^{\mu\nu}\Gamma^{\lambda}_{\mu\beta}\Gamma^{\beta}_{\nu\lambda} - g^{\mu\nu}\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\sigma}\right) \tag{11}$$

Where, κ is the Einstein constant. Let $\alpha = \sigma = r$ so that:

$$\kappa t^{r}_{r} = \frac{g^{rr}}{2}\Gamma^{\lambda}_{r\beta}\Gamma^{\beta}_{r\lambda} - g^{rr}\Gamma^{r}_{r\beta}\Gamma^{\beta}_{rr} + \frac{g^{tt}}{2}\Gamma^{\lambda}_{t\beta}\Gamma^{\beta}_{t\lambda} - g^{tt}\Gamma^{r}_{t\beta}\Gamma^{\beta}_{tr} \tag{12}$$

Then, substituting Greek indices by the Latin ones of the coordinate system - while applying the Einstein summation rule - and simplifying, we arrive at:

$$\kappa t_r^r = -\frac{g^{rr}}{2}\Gamma^r_{rr}\Gamma^r_{rr} + \frac{g^{rr}}{2}\Gamma^t_{rt}\Gamma^t_{rt} - \frac{g^{tt}}{2}\Gamma^r_{tr}\Gamma^r_{tr} + \frac{g^{tt}}{2}\Gamma^t_{tt}\Gamma^t_{tt} \tag{13}$$

Proceeding similarly, or simply by interchanging the indices r and t in the above equation, we arrive at the other energetic flux density of interest here:

$$\kappa t_t^t = -\frac{g^{tt}}{2}\Gamma^t_{tt}\Gamma^t_{tt} + \frac{g^{tt}}{2}\Gamma^r_{rt}\Gamma^r_{rt} - \frac{g^{rr}}{2}\Gamma^t_{rt}\Gamma^t_{rt} + \frac{g^{rr}}{2}\Gamma^r_{rr}\Gamma^r_{r} \tag{14}$$

Therefore:

$$t_t^t = -t_r^r \tag{15}$$

Recall:

$$diag\{t^\alpha_\sigma\} = [t^0_0, t^1_1, t^2_2, t^3_3] \tag{16}$$

Where, the first element in the square brackets is the energy density (00 component of the energetic fluxes), here equal to t_t^t. The other elements are normal stresses that, within the isotropic universe, are all equal to the pressure t_r^r. Therefore, considering gravity as a fluid, equation (15) implies that it has an equation of state (EoS) - the ratio of the pressure to the energy density - of $w = -1$.

4.0 HUBBLE EXPANSION IN A MODIFIED ROBERTSON-WALKER SPACETIME: IDENTIFICATION OF DARK ENERGY

Given the gravitational EoS derived above, we now seek to determine the nature of a gravitational mechanism that may drive the spatial expansion of the universe. To this end, we consider the gravitational field associated with a central gravitating body, or matter field.

In considerations of the static central field, the (Exterior) Schwarzschild metric has been very successful. Fixing its transverse coordinates so that $d\theta = d\phi = 0$, then the terms in which these infinitesimal coordinate vectors appear in Schwarzschild's linear element vanish and the spherically symmetric Schwarzschild metric is restricted to being radially symmetric. Now, by applying the Hubble scale factor a in its g_{rr} metric component, we produce a radially symmetric version of the gravitationally perturbed Robertson-Walker metric [8]. The scale factor is a function of coordinate time. The resulting metric and its inverse have the following components:

$$g_{rr} = -a^2(1 - \frac{r_s}{r})^{-1} \tag{17}$$

$$g^{rr} = -a^{-2}(1 - \frac{r_s}{r}) \tag{18}$$

$$g_{tt} = (1 - \frac{r_s}{r}) \tag{19}$$

$$g^{tt} = (1 - \frac{r_s}{r})^{-1} \tag{20}$$

The coordinate first derivatives of the components of the metric are:

$$\frac{\partial g_{rr}}{\partial x^r} = \frac{a^2 r_s}{r^2}(1 - \frac{r_s}{r})^{-2} \tag{21}$$

$$\frac{\partial g_{rr}}{\partial x^t} = -2a\dot{a}(1 - \frac{r_s}{r})^{-1} + \frac{a^2 \dot{r} r_s}{r^2}(1 - \frac{r_s}{r})^{-2} \tag{22}$$

$$\frac{\partial g_{tt}}{\partial x^r} = \frac{r_s}{r^2} \tag{23}$$

$$\frac{\partial g_{tt}}{\partial x^t} = \frac{r_s \dot{r}}{r^2} \tag{24}$$

4.1 Energetic Gravitational Fluxes in an Expanding Universe

Substituting from equations (17) to (20) into equations (5) to (**??**) and, subsequently, from the latter set and into equation (13), leads directly to:

$$\kappa t_r^r = -\frac{\dot{a}^2}{2a^2}(1 - \frac{r_s}{r})^{-1} + \frac{\dot{a}\dot{r} r_s}{2ar^2}(1 - \frac{r_s}{r})^{-2} \tag{25}$$

By means of the Hubble-Lemai tre law, we may replace the quotient $\dot{a}/a$ by the Hubble parameter – H. Then, so that the energy tensors of the gravitating body determining the Schwarzschild radius - $r_s = 2GM/c^2$ - do not change due to Hubble flow, we ensure that the observer moves with it away from the galactic centre - that serves as the origin of the coordinate chart here applied - at the radial speed of:

$$\dot{r} = \frac{dx^r}{dx^t} = \frac{1}{c}\frac{dr}{dt} = Hr \tag{26}$$

Where:

$$x^t = ct \tag{27}$$

So, with these substitutions, equation (25) becomes:

$$\kappa t_r^r = -\frac{H^2}{2}(1 - \frac{r_s}{r})^{-1} + \frac{H^2 r_s}{2r}(1 - \frac{r_s}{r})^{-2} \tag{28}$$

Figure 1 shows the radial profile of the gravitational pressure.

So, beyond the event horizon of the super massive black hole (SMBH), where $r > r_s$ defines the domain of application of the Exterior Schwarzschild metric, the pressure turns out to be negative and directly proportional to the square of the Hubble parameter. Clearly, this negative pressure is intimately related to the Hubble expansion. Both are signal features associated with the hypothetical dark energy of the cosmological constant - Λ. Figure 1 illustrates the initially rapid rise of the magnitude of the negative pressure as the distance from the SMBH increases and its fairly rapid flattening thereafter. At very large radii, the negative pressure approaches its maximum magnitude of - $H^2/2\kappa$. This maximum pressure is independent of r_s. That is, the maximum magnitude of the gravitational pressure - and therefore also that of the gravitational energy density - of every body, even of all bodies taken together, is the same. Such a saturation of the gravitational

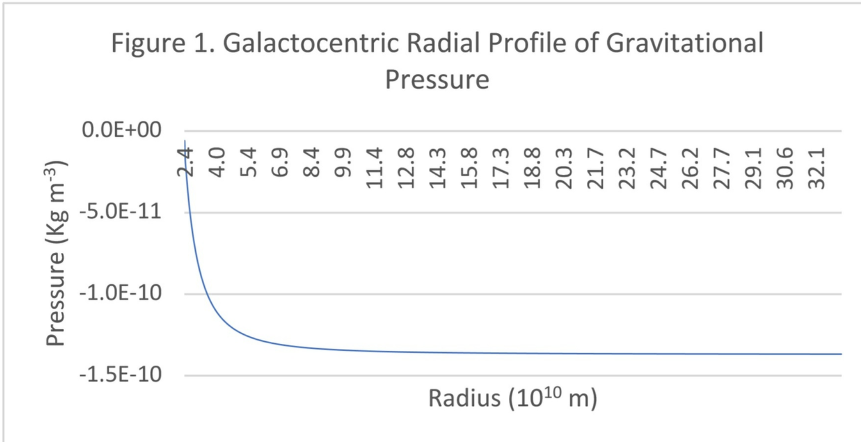

Figure 1: .

pressure (and energy density) occurring remotely from gravitating bodies of all scales of mass-energies and densities is remarkable.

The EoS of gravity revealed in equations (13), (14) and equation (15) imply that, without this continuous negative gravitational pressure and spatial expansion, the gravitational field and so space also would collapse. This suggests that gravitational pressure causes the spatial expansion.

4.2 Conservation of Energy in an Expanding Universe: a Derivation of the Hubble-Lemaitre law

Therefore, I propose that a growing gravitational field causes the expansion of space in a cooperation that preserves adherence to the energy conservation laws of gravity that apply at infinitesimal scales in the vacuum and of the total system - the latter including the gravitating matter and the gravitational field – expressed by [4], respectively, as:

$$\frac{\partial t^{\alpha}_{\sigma}}{\partial x^{\alpha}} = 0 \tag{29}$$

$$\frac{\partial}{\partial x^{\alpha}}(T^{\alpha}_{\sigma} + t^{\alpha}_{\sigma}) \tag{30}$$

Where, T^{α}_{σ} is the energy tensor of the gravitating body.

This cooperation may only occur if the incrementing energy only develops in emergent fields that come into being in infinitesimal emergent spaces and at the same energy density as that of the infinitesimal regions of the pre-existent field contiguous to the infinitesimal emergent spaces. Since, such a process of spatial expansion occurs at all radii greater than r_s, then the expansion of space occurs as a smooth interstitial emergence of new spaces throughout the spatial manifold. This may only occur if space is the form of the gravitational field. This notion is buttressed by the fundamental metrical coupling of the field and space expressed by equations (1) and (4).

So, since the gravitational energy density – with its magnitude, everywhere, being equal to that of the gravitational pressure - is conserved, then a gravitationally driven expansion would be locally isobaric.

All isobaric expansions, thermodynamic or cold, require the injection of energy. In thermodynamics, this energy is in the form of heat. In the cold gravitational isobaric expansion, emergent gravitational energy - that by virtue of the constraint of the conservation of energy density, may only occur in emergent spaces - performs the same role as heat does in isobaric thermodynamic expansions.

Consider the infinitesimal isobaric expansion of space in the form of a spherical shell of inner radius r and thickness dr. We propose that this expansion of space is due to the work that is done by the gravitational pressure within the infinitesimal region of the shell of volume dV. Conservation of energy implies that $dW + dU = 0$, where dW is the work done by the gravitational pressure and dU is gravitational energy emerging in the simultaneously emergent space, solely as a result of the gravitational pressure.

The work dW is the product of the pressure, the area of the inner surface of the shell - $4\pi r^2$ and the displacement and is expressed here as:

$$dW = 4\pi r^2 t_r^r dr \tag{31}$$

The volume of the sphere and the new infinitesimal spatial volume created by the gravitational pressure is the volume of the shell given as:

$$V = \frac{4}{3}\pi r^3 \tag{32}$$

and, by way of the radial derivative of the volume, the volume of the shell is:

$$dV = 4\pi r^2 dr \tag{33}$$

Conservation of energy implies:

$$dU = -dW = -4\pi r^2 t_r^r dr \tag{34}$$

Therefore, with a substitution from (33), the energy density within the shell, by virtue of equation (15), is given by:

$$\rho = \frac{dU}{dV} = -\frac{4\pi r^2 t_r^r dr}{4\pi r^2 dr} = -t_r^r = t_t^t \tag{35}$$

This shows how the emergent space is created by gravitational pressure and simultaneously occupied by the incrementing gravitational energy at the pre-existent density. In this process, the incremental gravitational energy is created purely by the work constantly being performed by the gravitational pressure in creating emergent spaces. In this way, the expansion of space and the growth of the gravitational field combine to yield the conservation of gravitational energy, at infinitesimal scales, as first enunciated by [4].

Consider this expansion of space as it occurs in two separated spherical regions, each of arbitrary radius that comoves with the Hubble flow, located in gravitational fields possibly of different gravitational pressures. We desire, firstly, to compare the rates of fractional volumetric increases that result from these pressures.

The power per unit volume, isobarically applied, is directly proportional to the constant local pressure. So, we may write:

$$\frac{1}{V}\frac{dW}{dt} \propto t_r^r \tag{36}$$

Conservation of energy implies that the power - per unit volume - delivered by the gravitational pressure, in its locally isobaric expansion of space, is given by:

$$\frac{1}{V}\frac{dW}{dt} = \frac{t^r_r}{V}\frac{dV}{dt} \tag{37}$$

Denoting the two scenarios of the expanding spheres by the indices 1 and 2, respectively, the two relationships above allow us to write:

$$\frac{t^r_{r,2}}{t^r_{r,1}} = \frac{\frac{1}{V_2}\frac{dW_2}{dt}}{\frac{1}{V_1}\frac{dW_1}{dt}} = \frac{\frac{t^r_{r,2}}{V_2}\frac{dV_2}{dt}}{\frac{t^r_{r,1}}{V_1}\frac{dV_1}{dt}} \tag{38}$$

Which first yields:

$$\frac{1}{V_1}\frac{dV_1}{dt} = \frac{1}{V_2}\frac{dV_2}{dt} \tag{39}$$

So, the fractional volumetric expansion rates are equivalent. With substitutions from (32) and (33), we get:

$$\frac{1}{r_1}\frac{dr_1}{dt} = \frac{1}{r_2}\frac{dr_2}{dt} = H \tag{40}$$

Therefore, H remains the same - irrespective of the strength of the gravitational field and the separation involved - and, thereby, also of location and time, due solely to the isobaric nature of the expansion of the invisible gravitational field underlying the observable spatial expansion. This affirms the uniformity and isotropy of the Hubble expansion across all distance and time scales and, thereby, confirms the nature of the Hubble parameter as a cosmic constant.

Thereby, this account identifies dark energy as the expanding gravitational field. In so doing, we have also shown that space is the form of the gravitational field.

5.0 INFLUENCES OF THE EXPANDING ENERGY FIELDS OF GRAVITY

5.1 Galactic Orbits and the Gravitational Field

In this theoretical framework, the acceleration of bodies in free fall is directly due to the gravitational field, (as opposed to the direct attraction of the gravitating body, as in Newtonian physics). So, here the satellite's acceleration is given by the geodesic equation:

$$\frac{d^2x^\tau}{ds^2} = \Gamma^\tau_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} \tag{41}$$

In the absence of information on 3-velocities, we may first extract the acceleration by ensuring that the observer comoves with the test body by setting:

$$\frac{dx^\mu}{ds} = \frac{dx^\nu}{ds} = 0 \quad : \mu, \nu = 1, 2, 3 \tag{42}$$

With these conditions, equations (1) and (14) yield:

$$\frac{dx^\mu}{ds} = \frac{dx^\nu}{ds} = \surd(g_{tt}) \approx 1 \quad : r \gg r_s, \mu, \nu = 0 \tag{43}$$

So, equation (31) yields:

$$\frac{d^2x^\tau}{ds^2} \approx \Gamma^\tau_{tt} \quad : \tau = 1, 2, 3 \tag{44}$$

These components of the 3-acceleration may be vectorially summed as:

$$\frac{d^2x^r}{ds^2} \approx \Gamma^r_{tt} \tag{45}$$

With the acceleration so determined, then substituting from equations (18), (22), and (9) into the above equation, the radial acceleration may be expressed as:

$$\frac{d^2x^r}{ds^2} \approx \Gamma^r_{tt} = -\frac{r_s}{2a^2r^2}(1 - \frac{r_s}{r}) \tag{46}$$

Now, the orbits of the satellites of many galaxies are approximately circular, so the (averaged) speed of rotation is given by the square root of the product of the acceleration and the radius, expressed here as:

$$v_{rot} \approx |r\Gamma^r_{tt}|^{1/2} = [\frac{c^2r_s}{2a^2r}(1 - \frac{r_s}{r})]^{1/2} \tag{47}$$

Where c is the speed of light that appears here, by virture of equation (27), in order to convert the natural time unit of measurement - equivalent to c seconds - to the time unit of the second. For $a = 1$, Figure 2 shows the profile of a rotation curve in the vicinity of the galactic centre obtained by means of equation (47).

(Orbits of solar and planetary systems do not approach the event horizons - that are greatly exceeded by the extents of the stars and planets - as closely as those of galaxies with SMBHs enclosed by their event horizons, so their declining rotation curves appear Kleperian.)

However, exceptions abound. We propose that certain exceptions are due to the growth of the energy fields of gravity in the vicinities of galaxies. These include unexpected gravitational effects that are currently attributed to dark matter.

So, instead of directly considering the mass-energy of the gravitating bodies in order to understand the strange kinematics of their satellites, we now seek to understand the effects of the growth of the gravitational field on these orbits. For this purpose, we consider the gravitational energy density field obtained from equations (15) and (28) as:

$$\kappa t^t_t = \frac{H^2}{2}[(1 - \frac{r_s}{r})^{-1} - \frac{r_s}{r}(1 - \frac{r_s}{r})^{-2}] \tag{48}$$

The profile of the energy density field is shown in Figure 3. The gravitational energy density is positive beyond r_s. Note the high and flat energy density of regions remote from the centre. The strength of the energy density field dramatically declines as the galactic centre is approached.

As the gravitational field evolves, the resultant motions of the satellites progressively become more remotely connected to the gravitating bodies that initially gave rise to the field. Therefore, unexpected velocities may be due to unexpected behaviour of the gravitational field in intragalactic regions.

5.2 The Accelerated Expansions of Space and of the Gravitational Field

The acceleration of the expansions of space and the gravitational field may be arrived at by differentiating equation (26) to yield:

$$\ddot{r} = H\dot{r} = H^2r \tag{49}$$

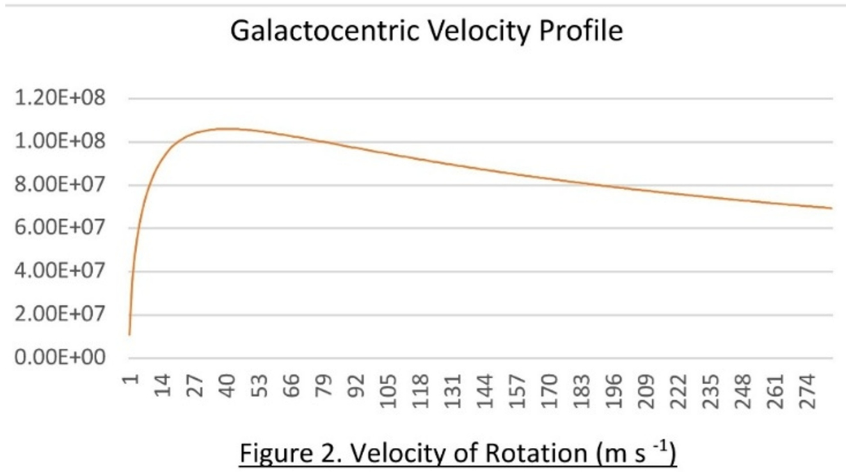

Figure 2: .

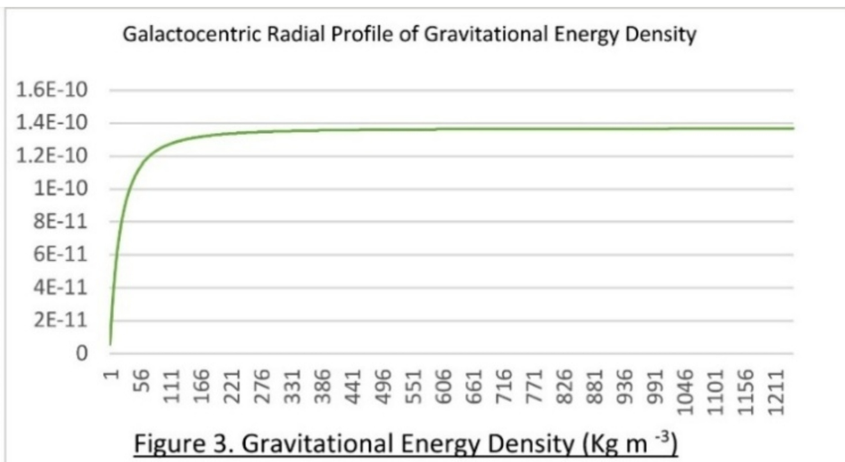

Figure 3: .

Which we get since H is a constant. So, two observer, while being separated only by the Hubble flow, will see each other in an accelerating recession.

Now, substitute from equation (27) into equation (26), rearrange, and then integrate:

$$\int_{r_0}^{r} \frac{dr}{r} = ln(\frac{r}{r_0}) = cH \int_{0}^{t} dt = cHt \tag{50}$$

So, we arrive at:

$$r = r_0 e^{cHt} \tag{51}$$

This equation describes the exponential expansion of space and the gravitational field over time (measured in seconds) corresponding to a constant Hubble parameter, that is, to a Hubble constant. The exponential term given in the equation above is the cosmic scale factor:

$$a(t) = e^{cHt} \tag{52}$$

So, the Hubble constant remains $H = (da/dx^t)/a = \dot{a}/a$ and $a(t_0) = 1$ at time $t_0 = 0$.

5.3 Historical Forms of the Laws of Curvilinear Gravity: Identification of Dark Matter

Equation (51) also constitutes a coordinate transformation of the radius across time and space due only to Hubble flow. Furthermore, since space is the form of the gravitational field and the latter determines the velocities of rotation of orbits, then equation (51) specifies the space-time coordinate transformation that occurs due to Hubble flow and, thereby, applicable to the radial variable in equations (47) and (48).

Consider an observer, initially at a distance r_0, who moves - in Hubble flow - away from the galactic centre so that his distance changes according to the historical relationship given in the advanced-time scaled coordinate transformation given in equation (50). So, over time, his distance from the galactic centre increases according to equation (50). However, the velocity of rotation of a satellite in his vicinity would not change as the gravitational field, at his comoving radius, conserves its energy density. So the total picture would seem irregular from the ahistorical point of view that is implied by the applications of equations (47) and (48).

Specifically, in these equations, only the radial variable would have changed in value, thereby rendering these equations inapplicable.

However, he may restore correct outcomes to his calculations by a coordinate transformation that, in these equations, substitutes the current radius r by its reversed-time scaled radius re^{-cHt} $(= r_0)$. So, generally, at any radius r, the following holds:

$$-\kappa t^r_r = \kappa t^t_t = \frac{H^2}{2}[(1 - \frac{r_s}{re^{-cHt}})^{-1} - \frac{r_s}{re^{-cHt}}(1 - \frac{r_s}{re^{-cHt}})^{-2}] \tag{53}$$

$$v_{rot} = [\frac{c^2 r_s}{2re^{-cHt}}(1 - \frac{r_s}{re^{-cHt}})]^{1/2} \tag{54}$$

Figure 4 shows, in a fixed galactocentric frame, a time-lapsed progression of development in a single galaxy. This series was produced by means of the applications of equations (53 and (54). Note the resemblance of the rotation curves to those of different galaxies. So, such galaxies may just be at different stages in a similar pattern of development.

The flattened rotation curves in the outer regions of numerous galaxies, resembling that shown in Figure 4d, have been attributed to a clustering dark matter field. However, these profiles have been produced by the expansion of the gravitational field. So, here, the expanding gravitational field performs the same role as does the hypothetical dark matter, (without the latter's challenges of identification and the 'cuspy' galactic cores of its mass-energy density profiles of its most prominent models.) This amounts to an identification of dark matter as being the expanding gravitational field.

In general, we may also calculate the velocity and gravitational energy densities that were observed prior to the observer reaching, in the Hubble flow, the radius r. In order to so do, in his equations (53) and (54),

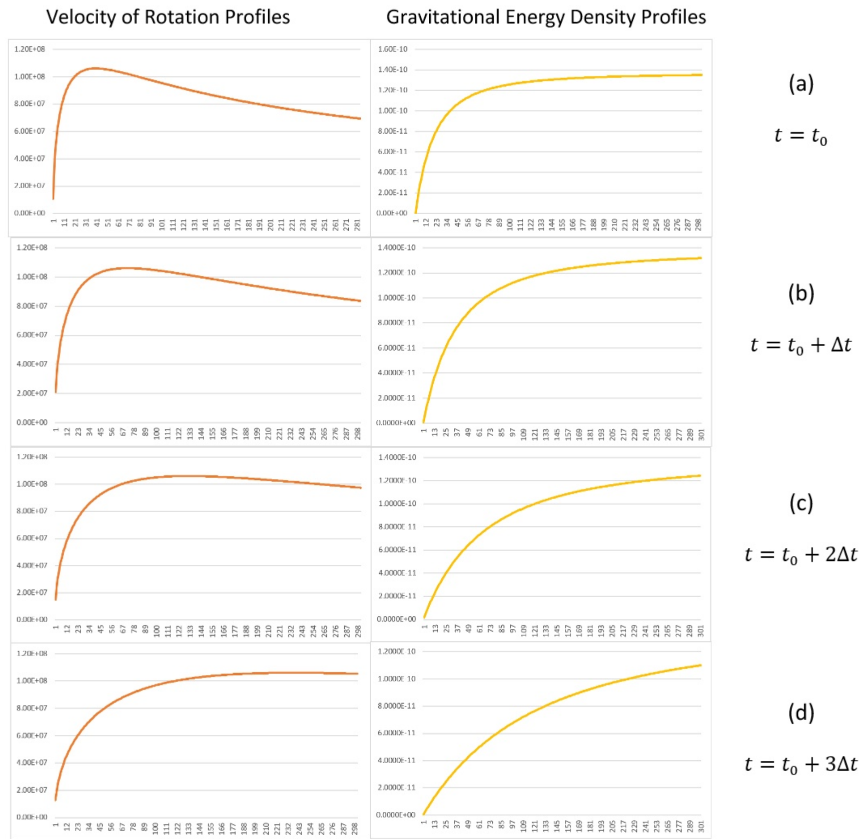

Figure 4. Evolution of Galactic Rotation Curve and the Gravitational Energy Density

Figure 4: .

the values of his times would be positive and smaller than his current one reflecting earlier periods after his time origin, or negative and so reflecting times earlier than his time origin $t_0 = 0$. We may also gain predictions of future orbits and energy densities by setting the time value greater than his current time. So, such equations give historical descriptions of unperturbed orbits and gravitational energy fields.

So, by means of the two equations above, we may obtain time profiles of development at fixed radii, as well as radial characteristics at fixed times - such as those of Figure 4 - of the expansion the orbital rotation curves and of the energetic fields of gravity and, thereby, of the kinematics of the bodies in them.

Now, orbits and gravitational lensing only occur in inherently curvilinear regions of the gravitational field [4]. So, 'dark matter halos' are simply regions of curvilinear gravity in the vicinities of matter fields. These regions extend well beyond the galactic orbits. In these 'halos', the density of gravitational energy is below its maximal values approached in voids.

The growth of the field extends the regions of gravitational curvilinearity, of the 'dark matter halo', as shown in Figure 4, leading to the notion of a 'clustering dark matter halo'.

5.4 Cosmic Voids and an Appproximately Flat Universe

The regions of gravitational curvilinearity - the 'dark matter halos' of enclosed matter fields - are embedded in a spatially dominant and approximately flat domain of the gravitational manifold. In these parts, in the limit - $r_s/r \rightarrow\ 0$, the equation of the line element obtained by substituting equations (19), (17), and (52) into equation (1) turns out to be that of *de* Sitter's:

$$(ds)^2 = (dx^0)^2 - e^{2cHt}[(dx^1)^2 + (dx^2)^2 + (dx^3)^2] \tag{55}$$

So, in these vast spaces, the metric approximates that of the flat spherically symmetrical expanding universe.Yet, it is curvilinear in galactic neighbourhoods. Therefore, the same metric applies across all distance scales. So, it avoids the challenges of integrating the curvilinear space-time of galaxies with a spatially flat space-time of the universe at large scales that confronted Einstein and Straus.

The flatness of space occurs beyond the 'dark matter halos' of galaxies, galactic clusters, and cosmic walls and in the great voids of gravitational flatness. So, the maximal intensity and flatness of the field, and not its Newtonian absence, prevail over the significantly greater portion of the observable universe consisting of large empty spaces. (This resolves the 'flatness problem' of the ΛCDM model.)

Equation (4) shows that the field components of gravity are functions of the coordinate derivatives of the components of the metric. So, an approximately constant metric in the void generally implies mostly vanishing gravitational field components. However, in this space-time, there is one space component of the gravitational field - Γ^r_{tr} – that, at large radii, approaches a constant non-zero value. Equations (19), (17), and (7) yield:

$$\Gamma^r_{tr} = -\frac{g^{rr}}{2}\frac{\partial g_{rr}}{dx^t} = -H + \frac{Hr_s}{2r}(1-\frac{r_s}{r})^{-1} \tag{56}$$

So:

$$\Gamma^r_{tr} = -H \quad : r_s/r\ \rightarrow\ 0 \tag{57}$$

It is this field component that gives rise to the approximately maximum value of gravitational energy density and pressure in these remote regions, as shown by equations (28) and (48).

Now, consider the geodesic 3-acceleration associated with this field component that is given by:

$$\frac{d^2x^r}{ds^2} = \Gamma^r_{rt}\frac{dx^r}{ds}\frac{dx^t}{ds} \tag{58}$$

Substitute from equations (26), (27), (49), (57), and $ds = cd\tau$, and rewrite (58) as:

$$\frac{d^2r}{d\tau^2} = -cH\frac{dr}{d\tau}\frac{dt}{d\tau} \tag{59}$$

Since, up to this point, the constant H has units of per c seconds, then converting to per second absorbs the constant c. So, substituting from (49):

$$\frac{d^2r}{d\tau^2} = -H\frac{dr}{dt}(\frac{dt}{d\tau})^2 = -\frac{d^2r}{dt^2}(\frac{dt}{d\tau})^2 \tag{60}$$

On the left of this equation is the acceleration, measured locally, due to the Hubble expansion. Locally, the coordinate radius r is equivalent to the proper distance $-D$. This is obtained in the time slice $dt = 0$ taken at the time of emission, $t = 0$, in the equation of the line element (55). So, making this substitution yields:

$$\frac{d^2D}{d\tau^2} = \frac{d^2r}{dt^2}(\frac{dt}{d\tau})^2 \tag{61}$$

This arranges the geodesic equation so that on the left is the local expansion, measured locally, and on the right is the same expansion measured from afar, as adjusted by a linear and homogenous (tensorial) transformation across time.

Consider the proper speed of recession measurable at the target and that is to be determined at the observer given as:

$$\frac{dD}{d\tau} = \frac{dr}{d\tau} = \frac{dr}{dt}\frac{dt}{d\tau} \tag{62}$$

In order to obtain the values of the proper separation's proper time derivatives, on the left of the two equations above, from the observed coordinate time derivatives of the radius on the right, the factor $dt/d\tau$ must be determined. This factor reflects the changes of the infinitesimal time ('tick') rate of the target's oscillator (source of electromagnetic radiation) as measured, usually from a distant, by an observer with an oscillator (clock) in his vicinity. This occurs because of the expansion of the wavelength as the radiation travels from the emitter to the observer. Since the wavelength lengthens, then the frequency reduces; redshifts.

Denote the parameters of the wave produced by the emitting oscillator by the subscript e and measurements of them by means of the observer's oscillator by o. Now, since the time rate is proportional to the period and thereby to the wavelength, then:

$$\frac{dt}{d\tau} = \frac{\lambda_o}{\lambda_e} = \frac{f_e}{f_o} = \frac{\lambda_o - \lambda_e}{\lambda_e} + 1 = z + 1 \tag{63}$$

Where f is the frequency and z is the fractional change in wavelength that occurs in transmission - the redshift.

Furthermore, as the wavelength changes due to the Hubble expansion, then:

$$1 + z = \frac{\lambda_o}{\lambda_e} = \frac{a(t_o)}{a(t_e)} = e^{cH(t_o - t_e)} \tag{64}$$

Where, equation (52) has been used to produce the last term above.

(In practice, the redshift z is determined spectroscopically. The separation is determined optically by relative luminosity of certain types of stars - 'standard candles' - or by the light curves of SN1a supernovae. The proper speed of the Hubble recession is obtained by means of the redshift and a cosmological model.)

So, it is the field component Γ^r_{tr} that, in the vast voids, governs the isotropic geodesic metric acceleration of the Hubble recession.

It must be kept in mind that, although being made clear here due to simplification of the metric, this field component acts in the same manner to drive the Hubble recession at all radii greater than r_s. However, within the regions of curvilinear gravity, its appearance is complexified by the gravitationally induced kinematical - as opposed to metrical - redshifting associated with the factor $(1 - r_s/r)$, that occurs to radiation emitted from gravitational wells. But, its own effects remain the same, in terms of the recessional rate it gives rise to. This is due to the invariance of the relative recessional rate due to its isobaric origins.

The attainment - in these volumetrically dominant voids - of maximal gravitational energy density, pressure, and, simultaneously, of flat space-time reflects the complexity and the importance of such regions in comprehending cosmic phenomena.

6.0 AUTO-INDUCTION: THE ENGINE OF GRAVITATIONAL EXPANSION

In presenting his theory of gravitation, Einstein developed his general field equations that represent the 'total system' - consisting, for example, of the massive gravitating body and its gravitational field - from his field equations of gravity 'in the absence of matter' - $R_{\mu\nu} = 0$ - and equation (4). So, he first arrives at the following relationship:

$$\frac{\partial}{\partial x^{\alpha}}(g^{\beta\sigma}\Gamma^{\alpha}_{\mu\beta}) = -\kappa(t^{\sigma}_{\mu} + \frac{\delta^{\sigma}_{\mu}}{2}t) \tag{65}$$

Then at this point, Einstein made the remarkable substitutions:

$$t \to t + T \quad t^{\sigma}_{\mu} \to t^{\sigma}_{\mu} + T^{\sigma}_{\mu} \tag{66}$$

Where, $t = t^{\mu}_{\mu} = trace\{t^{\sigma}_{\mu}\}$. Here, the trace is the sum of the elements along the diagonal of the pseudo-tensor as given in (16). So, as a result of its EoS, the trace of the energetic gravitational fluxes of gravity is given by:

$$t = 2t^{r}_{r} = -2t^{t}_{t} \tag{67}$$

By way of justification, Einstein offered the following:

'It must be admitted that this introduction of the energy-tensor of matter is not justified by the relativity postulate alone. For this reason, we have deduced it from the requirement that the energy of the gravitational field acts in the same way as other kinds of energy.' [Emphases added.]

So, by means of these substitutions, Einstein first arrives at:

$$\frac{\partial}{\partial x^{\alpha}}(g^{\beta\sigma}\Gamma^{\alpha}_{\mu\beta}) = -\kappa[(t^{\sigma}_{\mu} - \frac{\delta^{\sigma}_{\mu}}{2}) - (T^{\sigma}_{\mu} - \frac{\delta^{\sigma}_{\mu}}{2}T)] \tag{68}$$

And by such a pathway, Einstein arrived at his general field equations of gravitation.

With regards to the components of gravity, Einstein's substitutions amount to a feedback mechanism that drives continuous changes in the field components and, correspondingly, in the gravitational energy, even in the absence of changes in the energetic fluxes of matter. This is auto-induction of gravity.

In the exploration of the process of gravitational auto-induction, it is facilitative to ensure the exclusion of external factors by setting the energy tensors to zero, as they are in the vacuum. We may further simplify considerations by setting $\sigma = \mu$ in equation (65), whereby the Kronecker delta becomes $\delta_{m^{m=4.}}$ This yields:

$$\frac{\partial}{\partial x^{\alpha}}(g^{\beta\mu}\Gamma^{\alpha}_{\mu\beta}) = \kappa t \tag{69}$$

Rewrite as:

$$d(g^{\mu\beta}\Gamma^{\alpha}_{\mu\beta}) = \kappa t dx^{\alpha} \tag{70}$$

So, the components of the gravitational field that participate in time evolution at any - even at a fixed - radius r, are here identified by substituting a by t in the above equation. Then, integrating, we get:

$$\Delta(g^{\mu\beta}\Gamma^{t}_{\mu\beta})|^{ct}_{ct_0} = \kappa\int_{ct_0}^{ct} t dx^{t} \tag{71}$$

Where c is the speed of light and $x^t = ct$. Rewrite:

$$g^{\mu\beta}(r,t)\Gamma^t_{\mu\beta}(r,t) = g^{\mu\beta}(r_0,t_0)\Gamma^t_{\mu\beta}(r_0,t_0) + \kappa \int_{ct_0}^{ct} t dx^\alpha \tag{72}$$

Multiplying by the metric and, as $g_{\mu\beta}g^{\mu\beta} = \delta^\mu_\mu = 4$, we arrive at the auto-induced augmented field components:

$$\begin{aligned}\Gamma^t_{\mu\beta}(r,t) = \frac{1}{4}g_{\mu\beta}(r,t)g^{\mu\beta}(r_0,t_0)\Gamma^t_{\mu\beta}(r_0,t_0) + \\ \frac{1}{4}g_{\mu\beta}(r,t)\int_{ct_0}^{ct} t dx^\alpha\end{aligned} \tag{73}$$

This equation, that increments the field, is the critical regenerative phase of a positive gravitational feedback cycle: it describes the auto-induction of the time components of the gravitational field. However, such incremented field components have the same regenerative impact in the space components t^r_r of the energy fluxes as in their time components t^t_t, as is shown in equations (13) and (14). The updated field component above updates the energy density of equation (14) in its historical form given by equation (53), then the trace of the gravitational pseudo-tensor is updated by means of equation (67), whereby the cycle returns to the equation above to recommence.

So, as these field components evolve, so do the energetic fluxes, and - since the gravitational energy density is conserved - the field expands and, thereby, so does its form - space. By such a pathway, the auto-induced gravity results in an unconditional and continuous growth of the field, once it is seeded by the gravitating body.

6.1 Relative Mass-energy Density Weightings in the Induction of Gravity

Consider a spherical region of space undergoing the Hubble expansion. Denoting its initial conditions by the subscript 0, then with equation (51) and with Einstein's substitutions (66) into equation (69), there results the historical form of the equation of gravitational induction:

$$\frac{\partial}{\partial x^t}(g^{\mu\beta}\Gamma^t_{\mu\beta}) = \kappa[t + T_{B,0}e^{-3Hx^t} + T_{R,0}e^{-4Hx^t}] \tag{74}$$

Where, the subscripts B and R indicate baryonic matter and radiation, respectively. The Hubble expansion dilutes the mass-energy fields of radiation and baryons. So their induction of the gravitational field will be progressively reduced. For baryons, the dilution of energy density is directly proportional to the volume. For radiation, dilution occurs due to volumetric expansion as well as to its accompanying redshifting of the frequencies. However, the gravitational field preserves its density during spatial expansion

So, over time, gravitational energy increasingly dominates gravitational induction. Present estimates of 'dark energy' and 'dark matter' place the relative weight of gravitational energy density at approximately ninety-five percent.

7 .0 CONCLUSIONS

So, here stands a unitary identification of the dark energy and dark matter of the cosmos as being the singular energetic gravitational field. It was obtained solely by means of an orthodox development of, arguably, the most tested and successful physics theory of all time – general relativity.

New and significant aspects in this explanation include:

1. a single metric – the radially symmetric gravitationally perturbed Robertson-Walker metric - that:

- is applicable at all scales across the universe, smoothly accommodating descriptions in domains of both small and large scale phenomena.
- at large scales, recovers an FLRW line element, the one without curvature, that large-scale observations confirm. Here, in deep space, the metric appears as that of an empty *de* Sitter space-time.

2. The determination, in the space-time defined by this metric, of an equation of state (EoS) of the gravitational field as being: w=-1.

3. The identification of the mechanism of the auto-induction of gravity in Einstein's theory of general relativity.

4. The recognition that space is the form of the gravitational field.

5. The recognition that the expansion of gravity and its form - space - is due to the auto-induction of gravity and the nature of its EoS, under the constraints of conservation of gravitational energy density. The expansion is non-thermal and locally isobaric, with the continuous creation of new gravitational fields in the form of spaces occurring everywhere.

6. A unitary explanation of the cosmic phenomena attributed separately to dark energy and dark matter as being due to the actions of the auto-induced expanding gravitational field on embedded baryonic matter and radiation.

7. The demonstration that:

- the Hubble parameter, previously only empirically determined at large scales, is a cosmic constant. This is confirmed through the derivation, here, of the Hubble-Lemai tre law from the general conditions of gravitational expansion outlined in paragraph 5 above.
- the Hubble constant references a universal maximum magnitude, both of the gravitational pressure and of the gravitational energy density, expressed as $H^2/2\kappa$.

8 . The cosmic scale factor, irrespective of all types of gravitating bodies and radiation, is $a(t) = e^{cHt}$.

9. The recognition of the historical nature, and the development of the historical forms, of certain laws of nature including expressions for:

- the gravitational pressure and gravitational energy density.
- gravitational acceleration and the velocities of circular orbits.
- gravitational induction.

10. The recognition of the prime position of - the presently unidentified - gravitational energy in the equation of cosmic density parameters of cosmology and astrophysics.

8.0 DISCUSSION

The theoretical results obtained above strongly motivate development of techniques to advance empirical investigations to determine the linearity of the Hubble-Lemaitre at small scales $<$ 5 Mpc [9]This will test the proposal that the Hubble parameter is invariant.

It should be clear, in view of the account given here, that dark matter and dark energy are conceptions created to explain two sets of dynamical manifestations of one and the same thing – the auto-induced expansion of the gravitational field into new spaces that it creates. That is, the expansion of the gravitational field appears, through its different effects – the recession of galaxies and the unexpected orbits in galaxies and in their clusters – as being two different entities operating in mutually exclusive regions. These two sets of effects are manifestations of the actions of two coinciding energetic fluxes of gravity – pressure (dark energy) and energy density (dark matter). These fluxes are equal in magnitude everywhere.

(This avoids the 'cosmic coincidence problem' , confronting the ΛCDM model, of inexplicably comparable magnitudes of an increasing dark matter density and that of the constant dark energy density in this epoch.)

It must be noted that the 'model' of 'dark matter' presented here – that of the gravitational energy density field – has a galactocentric radial energy density profile that displays no 'cuspy' gradients in the vicinity of the galactic centre. In fact, as the centre is approached, the density declines, as do the predicted velocities of rotation. This decline of velocities, that challenges prominent models of dark matter, is generally empirically confirmed.

It is hoped that gravity will gain its place in scientific theory as a genuine type of energetic field, that is, one that also gravitates. Gravity deserves a place in the equation of cosmic mass-energy densities. Here, it will have the chance to display its impressive proportions as it performs its dominant and dialectical energetic actions that account for spatial expansion, large-scale spatial flatness, the structuring of matter, the null geodesics of radiation, and the small-scale curvilinear kinematics of the observable universe.

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