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RESEARCH

Stability of the Space Identification Problem for the Elliptic-Telegraph Differential Equation†

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Abstract

The present paper is devoted to study the space identification problem for the elliptic-telegraph differential equation in Hilbert spaces with the self-adjoint positive definite operator.

The main theorem on the stability of the space identification problem for the elliptic-telegraph differential equation is proved. In applications, theorems on the stability of three source identification problems for one dimensional with nonlocal conditions and multidimensional elliptic-telegraph differential equations are established.

KEYWORDS:
Identification problem, Elliptic-telegraph equation, Stability

1 | INTRODUCTION

Source identification problems for partial differential equations are used to model of biological, physical, system engineering and sociological processes and have been studied by many authors (see[1–12] and the references given therein).

Adavani and Biros[1] studied the fast algorithms for the solution of the source identification problem with linear elliptic partial differential equations constraints. The numerical techniques and the numerical experiments for the source identification problem with elliptic partial differential equations were constructed. Ashyralyev and Ashyralyyev[2] investigated the boundary value problem of determining the parameter of an elliptic equation in Banach space. Theorems of coercive stability estimates for the solution of boundary value problems for multi-dimensional elliptic equations were proved. Ashyralyev and Cekic[3] investigated the source identification problem for a telegraph equation with unknown parameter in a Hilbert space with the self-adjoint positive definite operator. Theorems of stability estimates for the solution of the telegraph equation were proved. In applications, three source identification problems for telegraph equations were obtained. The well-posedness of Neumann-type elliptic overdetermined problem with integral condition has been well established.[6] The author[7] proved the various estimates for the solution of the identification problem of inverse problem for the elliptic type equation. The stability, almost coercive stability, and coercive stability inequalities for its solution have been obtained. Katzourakis[10] investigated new methods of calculus of variations in L^∞ to study the ill-posed inverse problem of identifying the source of a non-homogeneous linear elliptic equation for Dirichlet conditions. Sabitov and Martem'yanova[11] studied the inverse problem for an equation of elliptic-hyperbolic type with a nonlocal boundary condition. Theorems of the uniqueness criterion and the stability of solutions with respect to the boundary value problem were proved. Avdonin and Nicaise[8] studied the source identification problems for the wave equation on graphs and the resolution of linear integral Volterra equations of the second kind for an interval. Theorems of the uniqueness and the existence of solutions were proved. Siskova and Slodicka[12] studied the inverse source problem in time-fractional wave differential equation with dynamical boundary condition for Neumann boundary conditions. Theorems of the uniqueness and existence of this solution were proved. The results of the numerical experiments were obtained.

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Various local and nonlocal boundary value problems for elliptic, hyperbolic, telegraph, hyperbolic- telegraph and elliptic-hyperbolic differential and difference equations and their applications have been investigated by many scientists (see (see [13–34] and the references given therein).

The nonlocal boundary value problems for hyperbolic-elliptic equation in a Hilbert space were studied, theorems on stability of this problem and the first and the second order of accuracy difference schemes for approximate solutions of this problem were proved.[14] Direk and Ashyraliyev[20] studied the initial-value problem for the integral-differential equation of the hyperbolic type in a Hilbert space. Theorems of the uniqueness of solvability of this problem were proved. The convergence estimates for the solutions of difference schemes were obtained. Gushchina[22] studied the equation of mixed elliptic-hyperbolic type in rectangular area with the conditions of periodicity and the nonlocal problem of A. A. Desin. Theorems of convergence of the constructed series in the class of regular solutions and the stability of the solution were proved. Ivanauskas, Novitski and Sapagovas[23] studied the stability of an explicit difference scheme for a linear hyperbolic equations with nonlocal integral boundary conditions. Theorem of the stability for a linear hyperbolic equations with nonlocal integral boundary conditions was proved. Mansour[26] studied the existence of traveling wave solutions for a hyperbolic-elliptic system of partial differential equations. Applied the geometric theory of singular perturbations. Theorem of the existence of the wave solution was proved. Sapagovas, Griskoniene and Stikoniene[28] applied the standard method of finite difference schemes for nonlinear elliptic equations with integral condition. Theorems of the convergence of all methods for this solution were proved. In application, the results of convergence between iterative methods were applied for the first time to nonlinear system. Sapagovas and Stikoniene[29] analyzed the generalization of the alternating-direction implicit method for the two-dimensional nonlinear elliptic equation with integral boundary condition in one coordinate direction. Theorem of the convergence of the iterative method was proved. Furthermore, the computational experiments results were obtained. Stikoniene, Sapagovas and Ciupaila[30] applied the iterative methods for the solution of the system of the difference equations derived from the elliptic equation with nonlocal conditions. Theorems on the convergence of faster iterative methods were proven. Novickij and Stikonas[31] studied the stability of a weighted finite difference scheme for wave equation with nonlocal boundary conditions. The linear hyperbolic equation with nonlocal integral boundary condition was investigated. The stability conditions in a special matrix norm were obtained.

However, source identification problems for elliptic-telegraph equations have not been well-investigated so far. Therefore, the main aim of this paper is to investigate the space identification problem for the elliptic-telegraph equation with parameter p .

Several identification problems for elliptic-telegraph equations can be reduced to the space source identification problem for the elliptic-telegraph equation

$$\begin{cases} \frac{d^2u(t)}{dt^2} + \alpha\frac{du(t)}{dt} + Au(t) = p + f(t), 0 < t < 1, \\ -\frac{d^2u(t)}{dt^2} + Au(t) = p + g(t), -1 < t < 0, \\ u(0) = \varphi, u_t(0^+) = u_t(0^-), u(-1) = \psi, u(1) = \xi \end{cases} \tag{1}$$

in a Hilbert space H with the self-adjoint positive definite operator $A \geq \delta I, \delta > 0$. Here p is the unknown parameter. The rest of the paper is organized as follows: In section 2, the main theorem on stability of problem (1) is established. In section 3, theorems on stability of three source identification problems for elliptic-telegraph equations are proved. Finally, section 4 is conclusion.

2 | THE MAIN THEOREM ON STABILITY

Denote that

$$u(t) = u(t; f(t), g(t), \varphi, \psi, \xi), \ \ p = p(f(t), g(t), \varphi, \psi, \xi).$$

By a solution of inverse problem (1) we mean a pair $(u(t), p)$ satisfying the following conditions:

1. The element $u(t)$ belong to D for all $t \in [-1, 1]$, and the function $Av(t)$ is continuous on $[-1, 1]$, $p \in H$. Here, $D = D(A)$ is the domain of an operator A.

2. $u(t)$ is twisly continuously differentiable on the interval $[-1, 1]$. The derivative at the endpoints of the interval are understood as the appropriate unilateral derivatives.

3. $(u(t), p)$ satisfies the evolution equation and local boundary conditions (1).

A solution of problem (1) defined in this manner will from now on be referred to as a solution of problem (1) in the space $C(H) \times H$. Here $C(H) = C([-1,1], H)$ is the space of continuous H-valued functions $u(t)$ defined on $[-1,1]$, equipped with the norm

$$||u||_{C(H)} = \max_{-1\le t\le 1} ||u(t)||_H . \tag{2}$$

In the present section, we will prove the main theorem on the stability of problem (1) in the space $C(H) \times H$.

To formulate our results we introduce the operator $G = A - \frac{\alpha^2}{4} I$. It is easy to see that for $\delta > \frac{\alpha^2}{4}$ G is the positive definite self-adjoint operator in the space H. Throughout, $\{c(t), t \geq 0\}$ is a strongly continuous cosine operator-function defined by the formula

$$c(t) = \frac{e^{itG^{1/2}} + e^{-itG^{1/2}}}{2}.$$

Then from the definition of the sine operator-function $s(t)$

$$s(t)u = \int_0^t c(y)u dy$$

it follows that

$$s(t) = G^{-1/2} \frac{e^{itG^{1/2}} - e^{-itG^{1/2}}}{2i}.$$

Now, let us give four lemmas that will be needed in the sequel.

Lemma 2.1. Assume that

$$\delta > \frac{\alpha^2}{4}, \alpha > 0. \tag{3}$$

Then for any $t \geq 0$, the estimates

$$\|c(t)\|_{H\to H} \leq 1, \left\|G^{1/2}s(t)\right\|_{H\to H} \leq 1, \tag{4}$$

$$\left\|B^{\beta} \exp\{-Bt\}\right\|_{H\to H} \leq 1, 0 \leq \beta \leq 1, \|(I - \exp\{-2B\})\|_{H\to H}^{-1} \leq M(\delta)$$

are satisfied. Here $B = A^{1/2}$.

Proofs of these estimates are based on the spectral representation of the self-adjoint positive definite operator in a Hilbert space.

Lemma 2.2. Assume that

$$\delta \geq \left(\frac{\alpha}{2}\right)^2 + 1, \alpha > 0.$$

Then, the operator

$$\left(I - e^{-\frac{\alpha}{2}}\left(c(1) + \frac{\alpha}{2}s(1)\right)\right)$$

has an inverse

$$E = \left(I - e^{-\frac{\alpha}{2}}\left(c(1) + \frac{\alpha}{2}s(1)\right)\right)^{-1}$$

and the following estimate holds

$$\|E\|_{H\to H} \leq \frac{1}{1 - \left(1 + \frac{\alpha}{2}\right)e^{-\frac{\alpha}{2}}}. \tag{5}$$

Proof. The proof of the estimate (5) is based on the estimate

$$\left\|c(1) + \frac{\alpha}{2}s(1)\right\|_{H\to H} \leq 1 + \frac{\alpha}{2}. \tag{6}$$

Using the definitions of $c(t)$ and $s(t)$ and positivity and self-adjointness property of A, we obtain

$$\left\|c(1) + \frac{\alpha}{2}s(1)\right\|_{H\to H} \leq 1 + \frac{\alpha}{2} \sup_{\delta\le\rho<\infty} \frac{1}{\left(\rho - \frac{\alpha^2}{4}\right)^{1/2}}$$

$$\leq 1 + \frac{\alpha}{2}\frac{1}{\left(\delta - \frac{\alpha^2}{4}\right)^{1/2}} \leq 1 + \frac{\alpha}{2}.$$

The proof of estimate (6) is completed. Lemma 2.2 is proved.

Lemma 2.3. Assume that

$$\left(\frac{\alpha}{2} + 1\right)^2 \geq \delta \geq \left(\frac{\alpha}{2}\right)^2 + 1, \alpha \geq 4. \tag{7}$$

Then, the operator

$$I - B\left(I + e^{-B}\right)^{-1}\left(I - e^{-B}\right) E e^{-\frac{\alpha}{2}} s(1)$$

has an inverse

$$Q = \left(I - B\left(I + e^{-B}\right)^{-1}\left(I - e^{-B}\right) E e^{-\frac{\alpha}{2}} s(1)\right)^{-1}$$

and the following estimate holds

$$||Q||_{H\to H} \le M(\alpha,\delta), \tag{8}$$

where $M(\alpha,\delta) > 0$.

Proof. We have that

$$Q = \left(I - e^{-\frac{\alpha}{2}}\left(c(1) + \frac{\alpha}{2}s(1)\right)\right)$$
$$\times\left\{I - e^{-\frac{\alpha}{2}}\left(c(1) + \frac{\alpha}{2}s(1)\right) - B\left(I + e^{-B}\right)^{-1}\left(I - e^{-B}\right) e^{-\frac{\alpha}{2}} s(1)\right\}^{-1}.$$

First, we will proof the estimate

$$\left\|c(1) + \frac{\alpha}{2}s(1) + B\left(I - e^{-B}\right)\left(I + e^{-B}\right)^{-1} s(1)\right\|_{H\to H} \le 1 + \frac{\alpha}{2} + \delta^{1/2}. \tag{9}$$

Using the definition of $s(t)$ and positivity and self-adjointness property of A and the triangle inequality, we obtain

$$\left\|A^{1/2}G^{-1/2}\right\|_{H\to H} \le \sup_{\delta\le\rho<\infty}\left(\frac{\rho}{\rho - \frac{\alpha^2}{4}}\right)^{1/2} \le \delta^{1/2} \tag{10}$$

and

$$\left\|B\left(I + e^{-B}\right)^{-1} s(1)\right\|_{H\to H} \le \sup_{\delta\le\rho<\infty}\left(\frac{\rho}{\rho - \frac{\alpha^2}{4}}\right)^{1/2}\left(\frac{1 - e^{-\rho^{1/2}}}{1 + e^{-\rho^{1/2}}}\right) \le \delta^{1/2}.$$

From that and from the estimate (6), it follows estimate (9). Using $\delta \le \left(\frac{\alpha}{2} + 1\right)^2$, we get

$$\left(1 + \frac{\alpha}{2} + \delta^{1/2}\right) e^{-\frac{\alpha}{2}} \le 2\left(1 + \frac{\alpha}{2}\right) e^{-\frac{\alpha}{2}}.$$

The proof of the estimate (8) is based on the estimate

$$2\sup_{4\le\alpha<\infty}\left(1 + \frac{\alpha}{2}\right) e^{-\frac{\alpha}{2}} < 1.$$

Denote

$$g(\alpha) = \left(1 + \frac{\alpha}{2}\right) e^{-\frac{\alpha}{2}}.$$

It is clear to see that

$$g'(\alpha) = -\frac{\alpha}{4} e^{-\frac{\alpha}{2}} < 0$$

for $\alpha > 0$. Therefore,

$$2\sup_{4\le\alpha<\infty}\left(1 + \frac{\alpha}{2}\right) e^{-\frac{\alpha}{2}} \le 2\left(1 + \frac{4}{2}\right) e^{-2} = \frac{6}{e^2} < 1.$$

Lemma 2.3 is proved.

Lemma 2.4. For the solution of problem (1) we have the following formula

$$u(t) = v(t) + A^{-1}p, \tag{11}$$

$$p = A\xi - Av(1), \tag{12}$$

where

$$v(t) = \left(I - e^{-2B}\right)^{-1}\left[\left(e^{tB} - e^{-(2+t)B}\right) v_0 + \left(e^{-(1+t)B} - e^{-(1-t)B}\right) v_{-1}\right.$$
$$\left. - \left(e^{-(1+t)B} - e^{-(1-t)B}\right)(2B)^{-1}\int_{-1}^{0}\left(e^{-(1+y)B} - e^{-(1-y)B}\right) g(y)dy\right]$$

$$+(2B)^{-1}\int_{-1}^{0}\left(e^{-|-t+y|B}-e^{(t+y)B}\right)g(y)dy,-1\leq t\leq 0 \tag{13}$$

and

$$v(t)=e^{-\frac{\alpha}{2}t}\left(c(t)+\frac{\alpha}{2}s(t)\right)v_0+e^{-\frac{\alpha}{2}t}s(t)v_0' \tag{14}$$

$$+\int_{0}^{t}e^{-\frac{\alpha}{2}(t-y)}s(t-y)f(y)dy,0\leq t\leq 1,$$

$$v_{-1}=v_0-\varphi+\psi, \tag{15}$$

$$v_0=E\left\{e^{-\frac{\alpha}{2}}s(1)v_0'+\int_{0}^{1}e^{-\frac{\alpha}{2}(1-y)}s(1-y)f(y)dy+\varphi-\xi\right\}, \tag{16}$$

$$v_0'=Q\left(I-e^{-2B}\right)^{-1}\left[B\left(I-e^{-B}\right)^{2}\left(I-e^{-\frac{\alpha}{2}}\left(c(1)+\frac{\alpha}{2}s(1)\right)\right)^{-1}\right.$$

$$\times\left\{\int_{0}^{1}e^{-\frac{\alpha}{2}(1-y)}s(1-y)f(y)dy+\varphi-\xi\right\}$$

$$\left.-2Be^{-B}(-\varphi+\psi)+\int_{-1}^{0}\left(e^{-(2+y)B}-e^{yB}\right)g(y)dy\right]. \tag{17}$$

Proof. We seek the solution of problem (1) by formula (11), where $v(t)$ is the solution of the following nonlocal boundary value problem

$$\begin{cases}\frac{d^2v(t)}{dt^2}+\alpha\frac{dv(t)}{dt}+Av(t)=f(t),0<t<1,\\[2mm] -\frac{d^2v(t)}{dt^2}+Av(t)=g(t),-1<t<0,\\[2mm] v(0)-v(-1)=\varphi-\psi,v(0)-v(1)=\varphi-\xi,v_t(0^+)=v_t(0^-)\end{cases} \tag{18}$$

for the differential equation in a Hilbert space H with self-adjoint positive definite operator A. Now, we will obtain the formula for the solution of nonlocal boundary value problem (18). It is known (see [4-5]) that for smooth data of the boundary value problems

$$\begin{cases}v''(t)+\alpha v'(t)+Av(t)=f(t),0<t<1,\\[2mm] v(0)=v_0,v'(0)=v_0',\end{cases} \tag{19}$$

$$\begin{cases}-v''(t)+Av(t)=g(t),-1<t<0,\\[2mm] v(0)=v_0,\quad v(-1)=v_{-1}\end{cases} \tag{20}$$

there are a unique solutions of the problems (19), (20) and formulas (13) and (14) hold. From nonlocal bondary condition $v_0-v_{-1}=\varphi-\psi$ it follows (15). Now, we obtain v_0. Applying (14) and condition $v_0-v(1)=\varphi-\xi$, we can write

$$e^{-\frac{\alpha}{2}}\left(c(1)+\frac{\alpha}{2}s(1)\right)v_0+e^{-\frac{\alpha}{2}}s(1)v_0'+\int_{0}^{1}e^{-\frac{\alpha}{2}(1-y)}s(1-y)f(y)dy=v_0-\varphi+\xi.$$

By Lemma 2, there exists the operator $E=\left(I-e^{-\frac{\alpha}{2}}\left(c(1)+\frac{\alpha}{2}s(1)\right)\right)^{-1}$ and the formula (16) holds. Now, we will obtain v_0'. Applying (13) and taking derivative at $t=0$ and using the condition $v_t(0^+)=v_t(0^-)$, we get

$$v_0'=\left(I-e^{-2B}\right)^{-1}\left[B\left(I+e^{-2B}\right)v_0-2Be^{-B}v_{-1}\right.$$

$$\left.+\int_{-1}^{0}\left(e^{-(2+y)B}-e^{yB}\right)g(y)dy\right]. \quad (21)$$

From that and formulas (15), (16) and (21), it follows that

$$v_0'=\left(I-e^{-2B}\right)^{-1}\left[B\left(I+e^{-2B}-2e^{-B}\right)\left(I-e^{-\frac{\alpha}{2}}\left(c(1)+\frac{\alpha}{2}s(1)\right)\right)^{-1}\right.$$

$$\times\left\{e^{-\frac{\alpha}{2}}s(1)v_0'+\int_0^1 e^{-\frac{\alpha}{2}(1-y)}s(1-y)f(y)dy+\varphi-\xi\right\}$$

$$\left.-2Be^{-B}(-\varphi+\psi)+\int_{-1}^{0}\left(e^{-(2+y)B}-e^{yB}\right)g(y)dy\right].$$

By Lemma 3, there exists the inverse operator

$$Q=\left(I-B\left(I+e^{-B}\right)^{-1}\left(I-e^{-B}\right)Ee^{-\frac{\alpha}{2}}s(1)\right)^{-1}$$

and the formula (17) holds. Therefore, for the formal solution of the problem (18) we have the formulas (13), (14), (15), (16) and (17). Formula for p follows from (11) and condition $u(1)=\xi$. Lemma 2.4 is proved.

Theorem 2.5. Suppose that $\varphi,\psi,\xi\in D(A)$, and $\alpha\geq 4$, $\left(\frac{\alpha}{2}+1\right)^2\geq\delta\geq\left(\frac{\alpha}{2}\right)^2+1$. Let $f(t)$ be continuously differentiable on $[0,1]$ and $g(t)$ be continuously differentiable on $[-1,0]$ functions. Then there is a unique solution of the problem (1) and the stability inequalities

$$\max_{-1\leq t\leq 1}\|u(t)\|_H+\left\|A^{-1}p\right\|_H\leq M(\alpha,\delta)\left[\|\varphi\|_H+\|\psi\|_H+\|\xi\|_H\right. \quad (22)$$

$$\left.+\max_{-1\leq t\leq 0}\left\|A^{-1/2}g(t)\right\|_H+\max_{0\leq t\leq 1}\left\|A^{-1/2}f(t)\right\|\right]_H,$$

$$\max_{-1\leq t\leq 1}\left\|\frac{d^2u(t)}{dt^2}\right\|_H+\max_{-1\leq t\leq 1}\|Au(t)\|_H+\|p\|_H$$

$$\leq M(\alpha,\delta)\left[\|A\varphi\|_H+\|A\psi\|_H+\|A\xi\|_H+\|g(0)\|_H\right. \quad (23)$$

$$\left.+\max_{-1\leq t\leq 0}\left\|g'(t)\right\|_H+\|f(0)\|_H+\max_{0\leq t\leq 1}\left\|f'(t)\right\|_H\right]$$

holds, where $M(\alpha,\delta)$ does not depend on $f(t), t\in[0,1], g(t), t\in[-1,0]$ and φ,ψ,ξ.

Proof. Applying formula (12), we can obtain estimates

$$\left\|A^{-1}p\right\|_H\leq\|\xi\|_H+\|v(1)\|_H\,,\|p\|_H\leq\|A\xi\|_H+\|Av(1)\|_H\,. \quad (24)$$

Therefore, by (11) we need to establish estimates for $\max\limits_{-1\leq t\leq 1}\|v(t)\|_H$, $\max\limits_{-1\leq t\leq 1}\|Av(t)\|_H$ and $\max\limits_{-1\leq t\leq 1}\left\|\frac{d^2v(t)}{dt^2}\right\|_H$. First, we obtain the estimate $\|v(t)\|_H$ for $-1\leq t\leq 1$ and the triangle inequality and estimates (3), (4), (5) and (8), we obtain (see [15])

$$\max_{-1\leq t\leq 0}\|v(t)\|_H\leq M_1(\alpha,\delta)\left[\left\|v_0\right\|_H+\left\|v_{-1}\right\|_H+\max_{-1\leq t\leq 0}\left\|A^{-1/2}g(t)\right\|_H\right] \quad (25)$$

Similarly, by (14) and the triangle inequality and estimates (3), (4), (5) and (8), we obtain (see [15])

$$\max_{0\leq t\leq 1}\|v(t)\|_H\leq M_2(\alpha,\delta)\left[\left\|v_0\right\|_H+\left\|A^{-1/2}v_0'\right\|_H+\max_{0\leq t\leq 1}\left\|A^{-1/2}f(t)\right\|_H\right]. \quad (26)$$

To end it we need estimates for $\left\|v_0\right\|_H$, $\left\|v_{-1}\right\|_H$ and $\left\|A^{-1/2}v_0'\right\|_H$. Using the triangle inequality and estimates (5), (8), (25) and (26), we get

$$\left\|v_0\right\|_H \leq \|E\|_{H\to H}\left\{e^{-\frac{\alpha}{2}}\left\|A^{1/2}G^{-1/2}\right\|_{H\to H}\left\|G^{1/2}s(1)\right\|_{H\to H}\left\|A^{-1/2}v_0'\right\|_H\right.$$
$$+\int_0^1 e^{-\frac{\alpha}{2}(1-y)}\left\|A^{1/2}G^{-1/2}\right\|_{H\to H}\left\|G^{1/2}s(1-y)\right\|_{H\to H}\left\|A^{-1/2}f(y)\right\|_H dy$$
$$\left.+\|\varphi\|_H+\|\xi\|_H\right\} \leq M_3(\alpha,\delta)\left[\|\varphi\|_H+\|\psi\|_H+\|\xi\|_H\right.$$
$$\left.+\max_{-1\leq t\leq 0}\left\|A^{-1/2}g(t)\right\|_H+\max_{0\leq t\leq 1}\left\|A^{-1/2}f(t)\right\|_H\right],$$
$$\left\|v_{-1}\right\|_H \leq \left\|v_0\right\|_H+\|\varphi\|_H+\|\psi\|_H$$
$$\leq M_4(\alpha,\delta)\left[\|\varphi\|_H+\|\psi\|_H+\|\xi\|_H\right.$$
$$\left.+\max_{-1\leq t\leq 0}\left\|A^{-1/2}g(t)\right\|_H+\max_{0\leq t\leq 1}\left\|A^{-1/2}f(t)\right\|_H\right]$$

and

$$\left\|A^{-1/2}v_0'\right\|_H \leq \|Q\|_{H\to H}\left\|\left(I-e^{-2B}\right)^{-1}\right\|_{H\to H}\left[\left\|\left(I-e^{-B}\right)^2\right\|_{H\to H}\|E\|_{H\to H}\right.$$
$$\times\left\{\int_0^1 e^{-\frac{\alpha}{2}(1-y)}\left\|A^{1/2}G^{-1/2}\right\|_{H\to H}\left\|G^{1/2}s(1-y)\right\|_{H\to H}\left\|A^{-1/2}f(y)\right\|_H dy\right.$$
$$\left.+\|\varphi\|_H+\|\xi\|_H\right\}+2\left\|e^{-B}\right\|_{H\to H}\left(\|\varphi\|_H+\|\psi\|_H\right)$$
$$\left.+\int_{-1}^0\left(\left\|e^{-(2+y)B}\right\|_{H\to H}+\left\|e^{yB}\right\|_{H\to H}\right)\left\|A^{-1/2}g(y)\right\|_H dy\right]$$
$$\leq M_5(\alpha,\delta)\left[\|\varphi\|_H+\|\psi\|_H+\|\xi\|_H\right.$$
$$\left.+\max_{-1\leq t\leq 0}\left\|A^{-1/2}g(t)\right\|_H+\max_{0\leq t\leq 1}\left\|A^{-1/2}f(t)\right\|\right]_H.$$

Therefore

$$\max_{-1\leq t\leq 1}\|v(t)\|_H \leq M_6(\alpha,\delta)\left[\|\varphi\|_H+\|\psi\|_H+\|\xi\|_H\right.$$
$$\left.+\max_{-1\leq t\leq 0}\left\|A^{-1/2}g(t)\right\|_H+\max_{0\leq t\leq 1}\left\|A^{-1/2}f(t)\right\|\right]_H. \tag{27}$$

Applying formula (11) and estimates (24) and (27) and the triangle inequality, we obtain estimate (22).

Second, we obtain the estimate $\|Av(t)\|_H$, for $-1\leq t\leq 1$. Using formulas (13) and (14) and integrating by parts, we can get formulas

$$Av(t)=\left(I-e^{-2B}\right)^{-1}\left[\left(e^{tB}-e^{-(2+t)B}\right)Av_0\right.$$
$$+\left(e^{-(1+t)B}-e^{-(1-t)B}\right)Av_{-1}-\frac{e^{-(1+t)B}-e^{-(1-t)B}}{2}$$
$$\left.\times\left\{-2e^{-B}g(0)+\left(I+e^{-2B}\right)g(-1)+\int_{-1}^0\left(e^{-(1+y)B}+e^{-(1-y)B}\right)g'(y)dy\right\}\right]$$
$$+(I+e^{2tB})g(t)-\frac{1}{2}(e^{-(1+t)B}-e^{-(1-t)B})g(-1) \tag{28}$$
$$-e^{tB}g(0)-\int_{-1}^0(e^{-(|y-t|B}+e^{(t+y)B})g'(y)dy, -1\leq t\leq 0,$$

and

$$Av(t)=e^{-\frac{\alpha}{2}t}\left(c(t)+\frac{\alpha}{2}s(t)\right)Av_0+e^{-\frac{\alpha}{2}t}s(t)Av_0'+AG^{-1}\left\{f(t)-e^{-\frac{\alpha}{2}t}c(t)f(0)\right.$$

$$+\int_0^t e^{-\frac{\alpha}{2}(t-y)}c(t-y)\left(\frac{\alpha}{2}f(y)+f'(y)\right)dy\Bigg\},0\leq t\leq 1. \tag{29}$$

Then, using (28) and estimates (3), (4), (5) and (8), we obtain (see [15])

$$\max_{-1\leq t\leq 0}\|Av(t)\|_H\leq M_7(\alpha,\delta)\left[\|Av_0\|_H+\|Av_{-1}\|_H\right. \tag{30}$$

$$\left.+\max_{-1\leq t\leq 0}\|g'(t)\|_H+\|g(0)\|_H\right].$$

Similarly, using (29) and estimates (3), (4), (5) and (8), we obtain (see [15])

$$\max_{0\leq t\leq 1}\|Av(t)\|_H\leq M_8(\alpha,\delta)\left[\|Av_0\|_H+\left\|A^{1/2}v_0'\right\|_H\right. \tag{31}$$

$$\left.+\|f(0)\|_H+\max_{0\leq t\leq 1}\|f'(t)\|_H\right].$$

To end it we need estimates for $\|Av_0\|_H, \|Av_{-1}\|_H$ and $\left\|A^{1/2}v_0'\right\|_H$. Using formulas (16) and (17) and integrating by parts, we can write the formulas

$$Av_0=E\Bigg\{e^{-\frac{\alpha}{2}}s(1)Av_0'+A\varphi-A\xi+AG^{-1}$$

$$\times\left(c(1)f(1)-c(1)f(0)-\int_0^1 e^{-\frac{\alpha}{2}(1-y)}c(1-y)\left[\frac{\alpha}{2}f(y)+f'(y)\right]dy\right)\Bigg\}$$

and

$$A^{1/2}v_0'=Q\left(I-e^{-2B}\right)^{-1}\Big[AG^{-1}\left(I-e^{-B}\right)^2E$$

$$\times\left(c(1)f(1)-c(1)f(0)-\int_0^1 e^{-\frac{\alpha}{2}(1-y)}c(1-y)\left[\frac{\alpha}{2}f(y)+f'(y)\right]dy\right)$$

$$+A\varphi-A\xi\}-2Be^{-B}\left(-A\varphi+A\psi\right)$$

$$\times\left\{2e^{-B}g(0)+\left(I+e^{-2B}\right)g(-1)+\int_{-1}^0\left(e^{-(2+y)B}-e^{yB}\right)g'(y)dy\right\}\Big].$$

Then, using the estimate (5), (8), (30) and (31), we obtain

$$\|Av_0\|_H\leq\|AE\|_{H\to H}\left\{e^{-\frac{\alpha}{2}}\left\|A^{3/2}G^{-1/2}\right\|_{H\to H}\left\|AG^{1/2}s(1)\right\|_{H\to H}\left\|A^{1/2}v_0'\right\|_H\right.$$

$$+\left\{\|f(1)\|_H+\|f(0)\|_H+\left(\|f(1)\|_H+\max_{0\leq t\leq 1}\|f'(t)\|_H\right)\right\}$$

$$\left.+\|A\varphi\|_H+\|A\xi\|_H\right\}\leq M_9(\alpha,\delta)\left[\|A\varphi\|_H+\|A\psi\|_H+\|A\xi\|_H\right.$$

$$\left.+\max_{-1\leq t\leq 0}\|g'(t)\|_H+\|g(0)\|_H+\max_{0\leq t\leq 1}\|f'(t)\|_H+\|f(0)\|_H\right],$$

$$\|Av_{-1}\|_H\leq\|Av_0\|_H+\|A\varphi\|_H+\|A\psi\|_H$$

$$\leq M_{10}(\alpha,\delta)\left[\|A\varphi\|_H+\|A\psi\|_H+\|A\xi\|_H+\max_{-1\leq t\leq 0}\|g'(t)\|_H\right.$$

$$\left.+\|g(0)\|_H+\max_{0\leq t\leq 1}\|f'(t)\|_H+\|f(0)\|_H\right]$$

and

$$\left\|A^{1/2}v_0'\right\|_H\leq\|Q\|_{H\to H}\left\|\left(I-e^{-2B}\right)^{-1}\right\|_{H\to H}$$

$$\times\left[\left\|\left(I-e^{-B}\right)^2\right\|_{H\to H}\|E\|_{H\to H}\right.$$

$$
\times\left\{\|f(t)\|_H+\|f(0)\|_H+\left(\|c(1)f(t)\|_H+\|c(1)\|_H\max_{0\le t\le 1}\left\|f'(t)\right\|_H\right)\right.
$$

$$
\left.+\|A\varphi\|_H+\|A\xi\|_H\right\}+2\left\|e^{-B}\right\|_{H\to H}\left(\|A\varphi\|_H+\|A\psi\|_H\right)
$$

$$
\left.+\left\{\left\|2e^{-B}\right\|_{H\to H}+\left\|A\left(I-e^{-2B}\right)^{-1}\right\|_{H\to H}\|g(-1)\|_H+\max_{-1\le t\le 0}\left\|g'(t)\right\|_H\right\}\right]
$$

$$
\le M_{11}(\alpha,\delta)\left[\|A\varphi\|_H+\|A\psi\|_H+\|A\xi\|_H+\max_{-1\le t\le 0}\left\|g'(t)\right\|_H\right.
$$

$$
\left.+\|g(0)\|_H+\max_{0\le t\le 1}\left\|f'(t)\right\|_H+\|f(0)\|_H\right].
$$

From these estimates and formulas (30) and (31) it follows

$$
\max_{-1\le t\le 1}\|Av(t)\|_H\le M_{12}(\alpha,\delta)\left[\|A\varphi\|_H+\|A\psi\|_H+\|A\xi\|_H+\max_{-1\le t\le 0}\left\|g'(t)\right\|_H\right.
$$

$$
\left.+\|g(0)\|_H+\max_{0\le t\le 1}\left\|f'(t)\right\|_H+\|f(0)\|_H\right]. \tag{32}
$$

Using estimates (24), (32), we obtain

$$
\|p\|_H\le\|A\xi\|_H+\left\|Av_1\right\|_H
$$

$$
\le M_{13}(\alpha,\delta)\left[\|A\varphi\|_H+\|A\psi\|_H+\|A\xi\|_H+\max_{-1\le t\le 0}\left\|g'(t)\right\|_H\right. \tag{33}
$$

$$
\left.+\|g(0)\|_H+\max_{0\le t\le 1}\left\|f'(t)\right\|_H+\|f(0)\|_H\right].
$$

Finally, applying the triangle inequality and equations (19) and (20) and estimate (32), we get

$$
\max_{-1\le t\le 1}\left\|\frac{d^2v(t)}{dt^2}\right\|_H\le M_{14}(\alpha,\delta)\left[\|A\varphi\|_H+\|A\psi\|_H+\|A\xi\|_H+\max_{-1\le t\le 0}\left\|g'(t)\right\|_H\right.
$$

$$
\left.+\|g(0)\|_H+\max_{0\le t\le 1}\left\|f'(t)\right\|_H+\|f(0)\|_H\right]. \tag{34}
$$

Estimate (23) follows from estimates (32), (33) and (34). Theorem 2.5 is proved.

3 | APPLICATIONS

In this section, we consider the applications of the Theorem 2.5.

First, we consider the equation

$$
\begin{cases}
u_{tt}(t,x)+\alpha u_t(t,x)-\left(a(x)u_x(t,x)\right)_x+\delta u(t,x)\\
=p(x)+f(t,x),\ 0<t<1,0<x<1,\\
-u_{tt}(t,x)-\left(a(x)u_x(t,x)\right)_x+\delta u(t,x)\\
=p(x)+g(t,x),\ \ -1<t<0,0<x<1.
\end{cases} \tag{35}
$$

Let $D=(-1,1)\times(0,1)$, $D_1=D\bigcap(t>0)$, $D_2=D\bigcap(t<0)$, $\mathfrak{I}=\{(t,x):t=0,0\le x\le 1\}$.

Problem. Find a pair of functions $(u(t,x),p(x))$ with the following properties:
1) $u(t,x)\in C(\overline{D})\bigcap C^1\left(D_1\bigcup D_2\bigcup\mathfrak{I}\right)\bigcap C^2\left(D_1\bigcup D_2\right)$,
2) $u(t,x)$ satisfies the equation (35) and the boundary conditions

$$
\begin{cases}
u(t,0)=u(t,1),\ u_x(t,0)=u_x(t,1),-1\le t\le 1,\\
u(0,x)=\varphi(x),u_t(0^+,x)=u_t(0^-,x),\\
u(-1,x)=\psi(x),u(1,x)=\xi(x),0\le x\le 1.
\end{cases} \tag{36}
$$

Problem (35) and (36) has a unique solution $(u(t,x),p(x))$ for the smooth functions $a(x)\geq a>0$, $a(1)=a(0), t\in(-1,1)$, $\delta,\alpha>0$, $\varphi(x),\psi(x),\xi(x),x\in[0,1]$. This allows us to reduce the boundary value problem (35) and (36) to the identification problem (1) in a Hilbert space $H=L_2[0,1]$ with a self-adjoint positive definite operator A^x defined by formula

$$A^x u(x)=-(a(x)u_x)_x+\delta u(x) \tag{37}$$

with domain

$$D(A^x)=\left\{u(x): u(x),u_x(x),(a(x)u_x)_x\in L_2[0,1],u(1)=u(0),u_x(1)=u_x(0)\right\}.$$

Applying the symmetry property of the space operator A^x with the domain $D(A^x)\subset W_2^2[0,1]$ and estimates (22) and (23) in $H=L_2[0,1]$, we can obtain the following theorem on stability of problem (35) and (36).

Theorem 3.1. Suppose that $\varphi,\psi,\xi\in W_2^2[0,1]$, and $\alpha\geq 4$, $\left(\frac{\alpha}{2}+1\right)^2\geq\delta\geq\left(\frac{\alpha}{2}\right)^2+1$. Let $f(t,x)$ be continuously differentiable in t on $[0,1]$ and $g(t,x)$ be continuously differentiable in t on $[-1,0]$ functions. Then the solutions of the identification problem (35) and (36) satisfy the stability estimates

$$\|u\|_{C([-1,1],L_2[0,1])}+\left\|(A^x)^{-1}p\right\|_{L_2[0,1]}\leq M_1(\alpha,\delta)\left[\|\varphi\|_{L_2[0,1]}+\|\psi\|_{L_2[0,1]}\right. \tag{38}$$
$$\left.+\|\xi\|_{L_2[0,1]}+\|f\|_{C([0,1],L_2[0,1])}+\|g\|_{C([-1,0],L_2[0,1])}\right],$$

$$\|u\|_{C^{(2)}([-1,1],L_2[0,1])}+\|u\|_{C([-1,1],W_2^2[0,1])}+\|p\|_{L_2[0,1]}$$
$$\leq M_2(\alpha,\delta)\left[\|\varphi\|_{W_2^2[0,1]}+\|\psi\|_{W_2^2[0,1]}+\|\xi\|_{W_2^2[0,1]}\right. \tag{39}$$
$$\left.+\|f\|_{C^{(1)}([0,1],L_2[0,1])}+\|g\|_{C^{(1)}([-1,0],L_2[0,1])}\right].$$

Here $M_1(\alpha,\delta)$ and $M_2(\alpha,\delta)$ do not depend on $\varphi(x),\psi(x),\xi(x),f(t,x)$ and $g(t,x)$.

Here, the Sobolev space $W_2^2[0,1]$ is defined as the set of all functions $u(x)$ defined on $[0,1]$ such that $u(x)$ and the second order derivative function $u''(x)$ are both locally integrable in $L_2[0,1]$, equipped the norm

$$\|u(x)\|_{W_2^2[0,1]}=\left(\int_0^1|u(x)|^2\,dx\right)^{\frac{1}{2}}+\left(\int_0^1\left|u_{xx}(x)\right|^2dx\right)^{\frac{1}{2}}.$$

Proof. Problem (35) and (36) can be written as abstract problem (1) in a Hilbert space $H=L_2[0,1]$ with self-adjoint positive definite operator $A=A^x$ defined by the formula (37). Here $f(t)=f(t,x),g(t)=g(t,x)$ and $u(t)=u(t,x)$ are known and unknown abstract functions with values in H and $p=p(x)$ is the unknown element of $L_2[0,1]$. Therefore, estimates (38) and (39) follow from estimates of Theorem 2.5. Theorem 3.1 is proved.

Second, let $\Omega\subset R^n$ be a bounded open domain with smooth boundary $S,\overline{\Omega}=\Omega\cup S$. In $[-1,1]\times\Omega$, we consider the boundary value problem for elliptic-telegraph equations

$$\begin{cases} u_{tt}(t,x)+\alpha u_t(t,x)-\sum\limits_{r=1}^{n}\left(a(x_r)u_{x_r}(t,x_r)\right)_{x_r} \\ =p(x)+f(t,x),\ 0<t<1,x=(x_1,...,x_n)\in\Omega, \\ -u_{tt}(t,x)-\sum\limits_{r=1}^{n}\left(a(x_r)u_{x_r}(t,x_r)\right)_{x_r} \\ =p(x)+g(t,x),\ \ -1<t<0,x=(x_1,...,x_n)\in\Omega, \\ u(0,x)=\varphi(x),u_t(0+,x)=u_t(0-,x), \\ u(-1,x)=\psi(x),u(1,x)=\xi(x),x\in\overline{\Omega}, \\ u(t,x)=0,x\in S,-1\leq t\leq 1 \end{cases} \tag{40}$$

is considered. Here $a_r(x)\geq a>0$, $(x\in\Omega)$, $\varphi(x),\psi(x),\xi(x)(x\in\overline{\Omega})$ and $f(t,x)$, $(t\in(0,1)),g(t,x),(t\in(-1,0))(x\in\Omega)$ and $(\delta>0)$ are given smooth functions.

We consider the Hilbert space $L_2(\overline{\Omega})$ of the all square integrable functions $u(x)$ defined on $\overline{\Omega}$, equipped with the norm

$$\|u(x)\|_{L_2(\overline{\Omega})} = \left(\int \cdots \int_{x\in\overline{\Omega}} |u(x)|^2 dx_1 \cdots dx_n\right)^{\frac{1}{2}}.$$

Problem (40) has a unique solution $(u(t,x), p(x))$ for the smooth functions $\varphi(x), \psi(x), \xi(x)$ and $a_r(x)$. This allows us to reduce the problem (40) to the boundary value problem (1) in the Hilbert space $H = L_2(\overline{\Omega})$ with a self-adjoint positive definite operator A^x defined by formula

$$A^x u(x) = -\sum_{r=1}^{n} (a_r(x) u_{x_r})_{x_r} \tag{41}$$

with domain

$$D(A^x) = \left\{u(x) : u(x), u_{x_r}(x), (a_r(x)u_{x_r})_{x_r} \in L_2(\overline{\Omega}), 1 \le r \le n, u(x) = 0, x \in S\right\}.$$

Theorem 3.2. Suppose that $\varphi, \psi, \xi \in L_2(\overline{\Omega})$, and $\alpha \ge 4$, $\left(\frac{\alpha}{2}+1\right)^2 \ge \delta \ge \left(\frac{\alpha}{2}\right)^2 + 1$. Let $f(t,x)$ be continuously differentiable in t on $[0,1]$ and $g(t,x)$ be continuously differentiable in t on $[-1,0]$ functions. Then the solutions of the identification problem (40) satisfy the stability estimates

$$\|u\|_{C\left(L_2(\overline{\Omega})\right)} + \left\|(A^x)^{-1}p\right\|_{L_2(\overline{\Omega})} \le M_3(\alpha,\delta)\Big[\|\varphi\|_{L_2(\overline{\Omega})} + \|\psi\|_{L_2(\overline{\Omega})} + \|\xi\|_{L_2(\overline{\Omega})} \tag{42}$$

$$+ \| f \|_{C([0,1],L_2(\overline{\Omega}))} + \| g \|_{C([-1,0],L_2(\overline{\Omega}))}\Big],$$

$$\|u\|_{C^{(2)}\left([-1,1],L_2[0,1]\right)} + \|u\|_{C\left([-1,1],W_2^2[0,1]\right)} + \left\|(A^x)^{-1}p\right\|_{L_2(\overline{\Omega})}$$

$$\le M_4(\alpha,\delta)\Big[\|\varphi\|_{L_2(\overline{\Omega})} + \|\psi\|_{L_2(\overline{\Omega})} + \|\xi\|_{L_2(\overline{\Omega})} \tag{43}$$

$$+ \| f \|_{C^{(1)}([0,1],L_2(\overline{\Omega}))} + \|g\|_{C^{(1)}([-1,0],L_2(\overline{\Omega}))} + \|f(0)\|_{L_2(\overline{\Omega})} + \|g(0)\|_{L_2(\overline{\Omega})}\Big],$$

where $M_3(\alpha,\delta)$ and $M_4(\alpha,\delta)$ do not depend on $\varphi(x), \psi(x), \xi(x), f(t,x)$and $g(t,x)$.

Here and in the future, the Sobolev space $W_2^2(\overline{\Omega})$ is defined as the set of all functions u defined on $\overline{\Omega}$ such that u and all second order partial differential derivative functions $u_{x_r x_r}, r = 1, , ..., n$ are both integrable in $L_2(\overline{\Omega})$, equipped with the norm

$$\|u\|_{W_2^2(\overline{\Omega})} = \|u\|_{L_2(\overline{\Omega})} + \left(\int \cdots \int_{x\in\overline{\Omega}} \sum_{r=1}^{n} \left|u_{x_r x_r}\right|^2 dx_1 \cdots dx_n\right)^{\frac{1}{2}}.$$

Proof. Problem (40) can be written as abstract problem (1) in a Hilbert space $H = L_2(\overline{\Omega})$ with self-adjoint positive definite operator $A = A^x$ defined by the formula (41). Here $f(t) = f(t,x), g(t) = g(t,x)$ and $u(t) = u(t,x)$ are known and unknown abstract functions defined on $\overline{\Omega}$ with values in $H = L_2(\overline{\Omega})$ and $p = p(x)$ is the unknown element of $L_2(\overline{\Omega})$. Therefore, estimates (42) and (43) follow from estimates of Theorem 2.5. Theorem 3.2 is proved.

Theorem 3.3. For the solution of the elliptic differential problem[35]

$$A^x u(x) = \mu(x), x \in \Omega, u(x) = 0, x \in S,$$

the following coercivity inequality holds

$$\sum_{r=1}^{n} \left\|u_{x_r x_r}\right\|_{L_2(\overline{\Omega})} \le M_5 ||\mu||_{L_2(\overline{\Omega})}.$$

Here M_5 does not depend on $\mu(x)$.

Third, in $[-1,1]\times\Omega$, the boundary value problem for the elliptic-telegraph equation

$$\begin{cases} u_{tt}(t,x)+\alpha u_t(t,x)-\sum\limits_{r=1}^{n}\left(a(x_r)u_{x_r}(t,x_r)\right)_{x_r}+\delta u \\ =p(x)+f(t,x),\ 0<t<1, \\ -u_{tt}(t,x)-\sum\limits_{r=1}^{n}\left(a(x_r)u_{x_r}(t,x_r)\right)_{x_r}+\delta u \\ =p(x)+g(t,x),\ \ -1<t<0, \\ x=(x_1,...,x_n)\in\Omega, \\ u(0,x)=\varphi(x),u_t(0+,x)=u_t(0-,x), \\ u(-1,x)=\psi(x),u(1,x)=\xi(x),x\in\overline{\Omega}, \\ \frac{du(t,x)}{d\vec{m}}=0,x\in S,-1\le t\le 1. \end{cases} \tag{44}$$

is considered. Here, $\vec{m}$ is the normal vector to S, $a_r(x)\ge a>0$, $(x\in\Omega)$, $\varphi(x),\psi(x),\xi(x)(x\in\overline{\Omega})$ and $f(t,x)$, $(t\in(0,1)), g(t,x),(t\in(-1,0))(x\in\Omega)$ and $(\delta>0)$ are given smooth functions.

Problem (44) has a unique solution $(u(t,x),p(x))$ for the smooth functions $\varphi(x),\psi(x),\xi(x)$ and $a_r(x)$. This allows us to reduce the problem (40) to the boundary value problem (1) in the Hilbert space $H=L_2(\overline{\Omega})$ with a self-adjoint positive definite operator A^x defined by formula

$$A^x u(x)=-\sum_{r=1}^{n}(a_r(x)u_{x_r})_{x_r}+\delta u \tag{45}$$

with domain

$$D(A^x)=\left\{u(x):u(x),u_{x_r}(x),(a_r(x)u_{x_r})_{x_r}\in L_2(\overline{\Omega}),1\le r\le n,\frac{\partial u\,(x)}{\partial\vec{m}}=0,x\in S\right\}.$$

Theorem 3.4. For the solutions of problem (40), we have following stability estimates

$$\begin{gathered}\|u\|_{C\left(L_2(\overline{\Omega})\right)}+\left\|(A^x)^{-1}p\right\|_{L_2(\overline{\Omega})} \\ \le M_6(\alpha,\delta)\left[\|\varphi\|_{L_2(\overline{\Omega})}+\|\psi\|_{L_2(\overline{\Omega})}+\|\xi\|_{L_2(\overline{\Omega})}\right. \\ \left.+\|\ f\ \|_{C([0,1],L_2(\overline{\Omega}))}+\|\ g\ \|_{C([-1,0],L_2(\overline{\Omega}))}\right],\end{gathered} \tag{46}$$

$$\begin{gathered}\|u\|_{C^{(2)}\left([-1,1],L_2[0,1]\right)}+\|u\|_{C\left([-1,1],W_2^2[0,1]\right)}+\left\|(A^x)^{-1}p\right\|_{L_2(\overline{\Omega})} \\ \le M_7(\alpha,\delta)\left[\|\varphi\|_{L_2(\overline{\Omega})}+\|\psi\|_{L_2(\overline{\Omega})}+\|\xi\|_{L_2(\overline{\Omega})}\right. \\ \left.+\|\ f\ \|_{C^{(1)}([0,1],L_2(\overline{\Omega}))}+\|g\|_{C^{(1)}([-1,0],L_2(\overline{\Omega}))}+\|f(0)\|_{L_2(\overline{\Omega})}+\|g(0)\|_{L_2(\overline{\Omega})}\right].\end{gathered} \tag{47}$$

where $M_6(\alpha,\delta)$ and $M_7(\alpha,\delta)$ do not depend on $\varphi(x),\psi(x),\xi(x),f(t,x)$and $g(t,x)$.

Proof. Problem (44) can be written in abstract form (1) in a Hilbert space $L_2(\overline{\Omega})$ with self-adjoint positive definite operator $A=A^x$ defined by the formula (45). Here $f(t)=f(t,x),g(t)=g(t,x)$ and $u(t)=u(t,x)$ are known and unknown abstract function defined on $\overline{\Omega}$ with values of $H=L_2(\overline{\Omega})$ and $p=p(x)$ is the element of $L_2(\overline{\Omega})$. Therefore, estimates (46) and (47) follow from estimates of Theorem 2.5. Furthermore, Theorem 3.4 is proved.

Theorem 3.5. For the solution of the elliptic differential problem[35]

$$\begin{cases} A^x u(x)=\mu(x),x\in\Omega, \\ \frac{\partial u(x)}{\partial\vec{m}}=0,x\in S, \end{cases}$$

the following coercivity inequality holds

$$\sum_{r=1}^{n} \left\| u_{x_r x_r} \right\|_{L_2(\overline{\Omega})} \leq M_8 ||\mu||_{L_2(\overline{\Omega})}.$$

Here M_8 does not depend on $\mu(x)$.

4 | CONCLUSIONS

In the present paper, the stability of the space identification problem for the elliptic-telegraph differential equation is discussed. The main theorem on the stability of the space identification problem for the elliptic-telegraph differential equation is established. In applications, the stability of three problems of the space identification problem for the elliptic-telegraph differential equations are obtained.

Two-step difference schemes for the numerical solutions of the identification problem for the elliptic-telegraph differential equation can be presented.[36] Of course, the stability estimates for the solution of this difference schemes can be obtained.

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