S.1 Discussion of figures related to climate feedback

Section 5.3 presents results of climate feedback calculations related to the revision to the water vapor continuum made in this study. Here we discuss information about these calculations unrelated to the continuum revision. This discussion has been inspired by and is complementary to many excellent recent studies on spectral aspects of climate feedback, including Feng et al. (2022), Koll et al. (2022), Jeevanjee et al. (2021), McKim et al. (2021), and Seeley and Jeevanjee (2020).

Detailed spectral information related to climate feedback considerations can be seen in Fig. 23, where panel (b) shows the spectral results underlying the climate feedback results shown as solid curves (for MT_CKD_4.2) for the various spectral regions in Fig. 22a, while panel (c) shows the spectral differences due to the revised continuum (corresponding to the results in Fig. 22b). (Analogous results to those presented in Fig. 23 can be seen in Fig. S4 for MT_CKD_4.2_closure.)

- For the lowest temperature range shown (240-250K, black/grey), Fig. 23b indicates that significant contributions to the climate feedback occur in the far-infrared (200-600 cm^{-1}) and infrared window. At these temperatures, the far-infrared is semi-transparent (Mlawer et al., 2019; Turner & Mlawer, 2010; Harries et al., 2008) and acts as a partial window, allowing some surface radiation to escape to space. As a result, there is a significant increase in the radiation emitted to space in both this region and the infrared window as the surface temperature increases.
- As the surface temperature increases to ~280K, the relative contributions of these two spectral regions to climate feedback is modified, with an increase in the window region and a decrease in the far-infrared. As shown in Fig. 22, these changes are of roughly equal magnitude and the total climate feedback stays fairly constant over this temperature range. As the temperature increases in this range, in the infrared window there is an increase in the climate feedback in the most transparent part of the region (950-1000, 1070-1300 cm^{-1}) since almost all of the surface radiation reaches the top of the atmosphere. In the less transparent part of the window (750-950 cm^{-1}), for surface temperatures above 260K the impact of the greater atmospheric absorption of the increasingly moist atmosphere begins to outweigh the increased surface radiation, and the climate feedback begins to decrease, although it is still quite significant. In contrast, for these surface temperatures the opacity of the far-infrared (200-600 cm^{-1}) is too

large (due to increased PWV values) to be considered a window and most of the surface radiation does not reach the TOA. As has been shown previously (e.g. Slingo and Webb, 1997, and Simpson, 1928), the outgoing radiation in the far-infrared under these conditions stays fairly constant with increasing surface temperatures since the impact of higher tropospheric temperatures is counterbalanced by the higher altitudes at which radiation emitted by the atmosphere can escape to space.

- As the surface temperature increases to 300K, the climate feedback across the entire infrared window decreases with surface temperature due to the rapidly increasing opacity due to self continuum absorption (proportional to square of water vapor abundance), although the climate feedback itself still remains positive throughout this region. This decrease is mitigated somewhat by increases in the climate feedback in the far-infrared, demonstrating that the counterbalancing mentioned above is only approximate, and in the CO_2 region (600-750 cm^{-1}). The behavior in the far-infrared can be seen in Fig. 23b, as well as in an expanded view in Fig. S12a, which demonstrates that the modest increase in climate forcing in this temperature regime occurs both in spectral locations dominated by absorption lines and in microwindows between lines. For one far-infrared microwindow, Fig. S13 shows the cumulative tropospheric optical depth downward from the tropopause as a function of layer temperature for the surface temperature values analyzed. The temperatures at which the cumulative optical depth reaches unity (i.e. roughly the temperature of emission to space) modestly increases with surface temperature, resulting in positive climate feedback values at this spectral location. The rate of this increase results in an increasing climate feedback for surface temperatures below ~305K, although it decreases at higher temperatures. The opaque region CO_2 region exhibits somewhat different behavior. In this region, the cumulative optical depth from the tropopause downward is a fairly constant function of pressure (not shown). Therefore, in the part of this region in which the outgoing radiation is from the troposphere (e.g. see 600-620 cm^{-1} in Fig. S12b), the positive climate feedback does not change too dramatically from 280-310K. In the more opaque part of the band (640-680 cm^{-1}), the climate feedback tends toward zero since the emission to space for all surface temperatures originates in the stratosphere, which is defined to be isothermal (at 220K) in these calculations.
- At surface temperatures just greater than 300K, the climate feedback in the more opaque part of the infrared window (750-1000 cm^{-1}) becomes negative while the more transparent region

(1070-1300 cm^{-1}) remains at a small positive value. When the surface temperature exceeds 310K, the entire window has negative climate feedback. As is the case for lower surface temperatures, the CO_2 region compensates for some of this decrease. As can be seen in Fig. S12b, most of this increase occurs in the center of this band (640-680 cm^{-1}), although it is unclear to what extent this calculated behavior is physically realistic since the isothermal stratosphere becomes quite thin (i.e. less air, so less CO_2) for higher surface temperatures, leading to increased emission to space from the troposphere. In the far-infrared, for these higher temperatures the climate feedback shows a slight decrease, with a climate feedback increase in more opaque parts of this region and a larger decrease in the less opaque regions, which have negative climate feedback for surface temperatures greater than 330K.

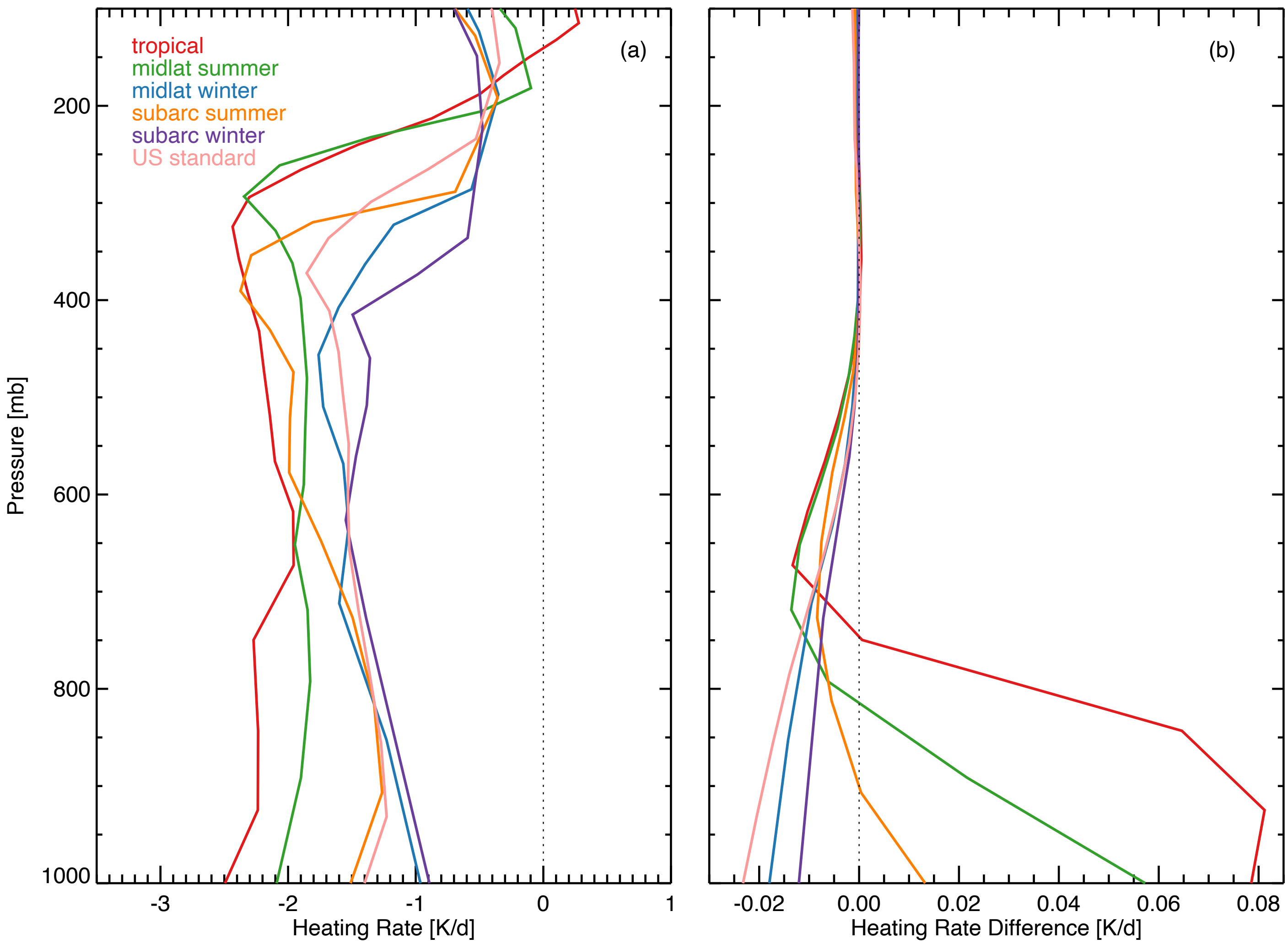

Fig. S1. For six standard atmosphere, (a) longwave heating rates from LBLRTM calculations using MT_CKD_4.1.1 and (b) difference in heating rates between calculations that use MT_CKD_4.2_closure and calculations that use MT_CKD_4.1.1. Comparable to Fig. 20.

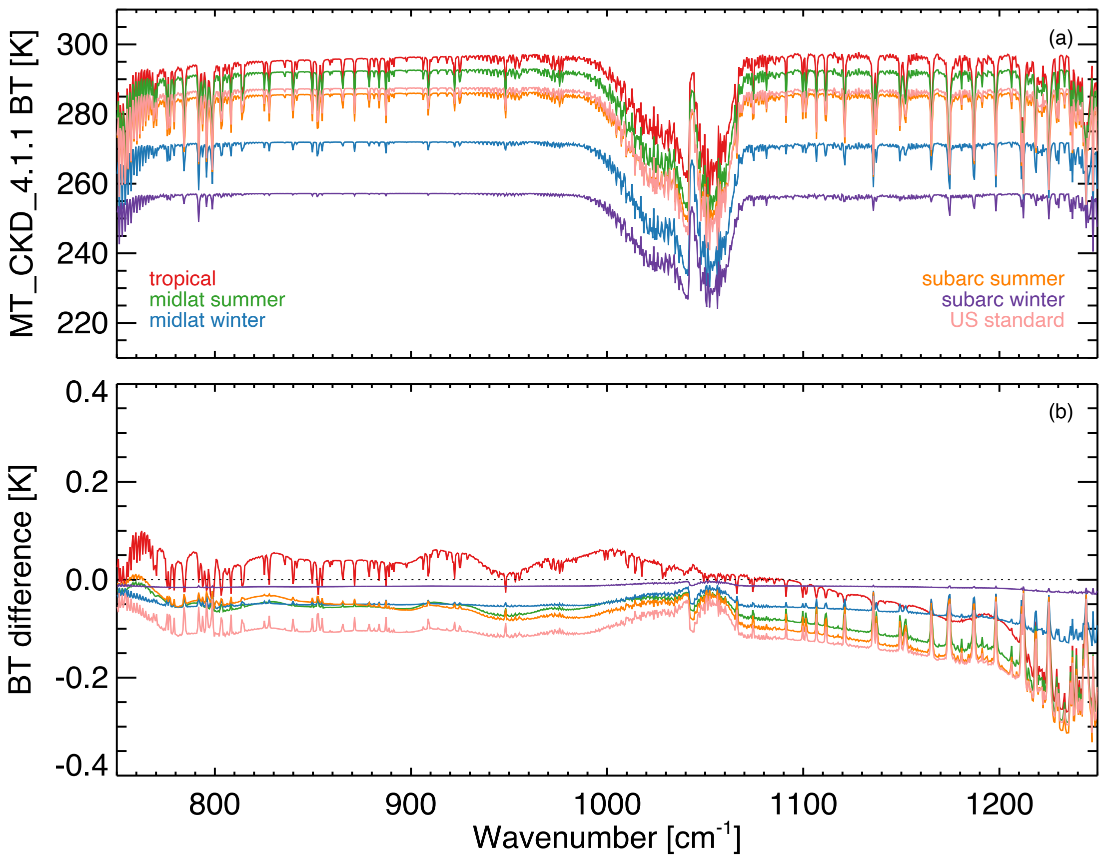

Fig. S2. For six standard atmospheres, (a) brightness temperatures calculated with LBLRTM with MT_CKD_4.1.1 and (b) brightness temperature differences between calculations that use MT_CKD_4.2_closure and calculations that use MT_CKD_4.1.1. Comparable to Fig. 21.

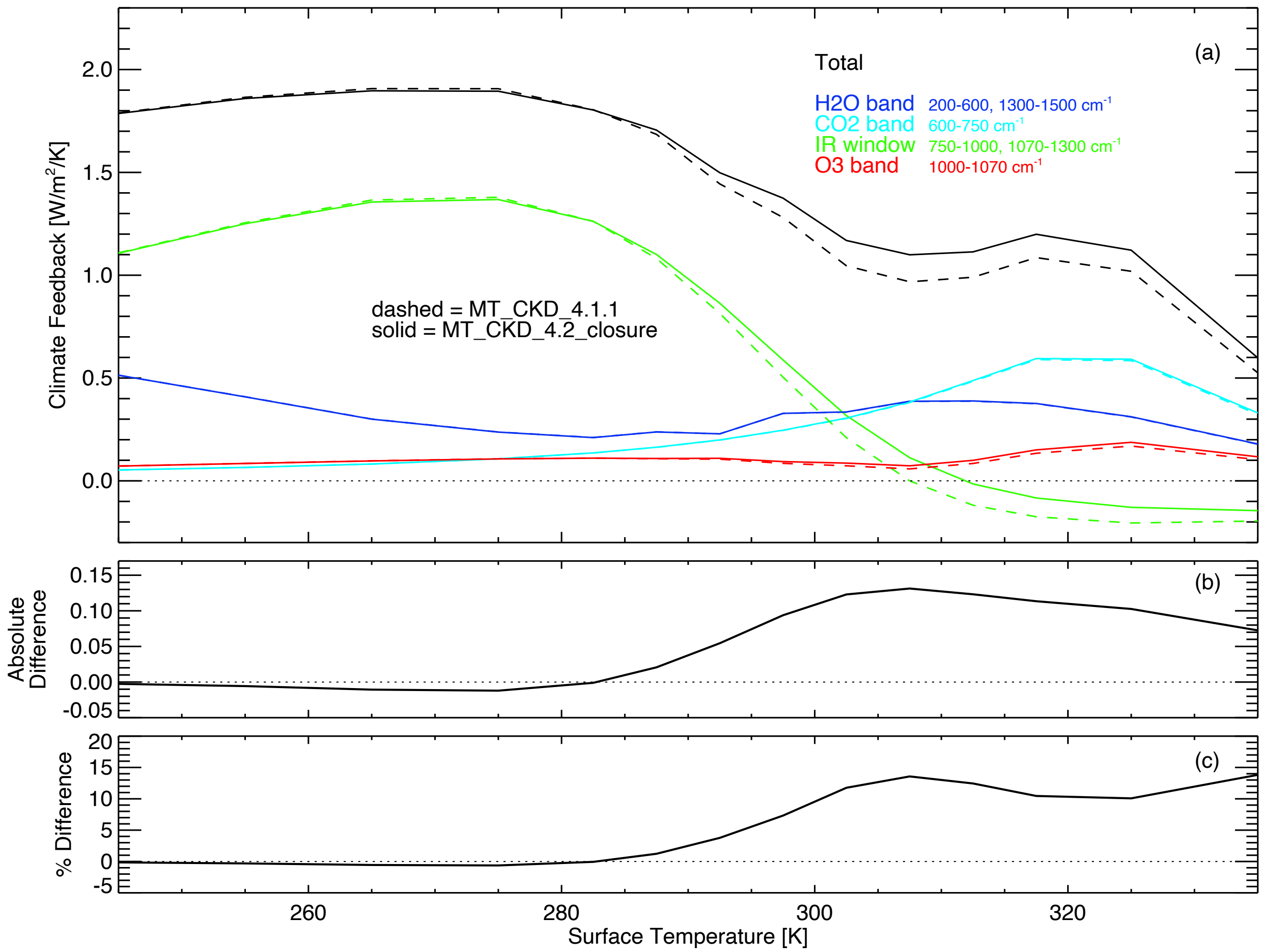

Fig. S3. As a function of surface temperature in moist adiabat profiles (as described in text): (a) climate feedback for full longwave region (black), water vapor absorption bands (red), CO_2 ν2 band (cyan), infrared window (green), and ozone band (red). Solid curves use revised continuum (MT_CKD_4.2_closure) in the calculations while dashed curves use previous continuum (MT_CKD_4.1.1); (b) for full longwave, climate feedback differences between calculations using MT_CKD_4.2_closure and MT_CKD_4.1.1, and (c) percentage differences in climate feedback between calculations using MT_CKD_4.2_closure and MT_CKD_4.1.1. Climate feedback is defined as the change in TOA flux per unit change in surface temperature. Comparable to Fig. 22.

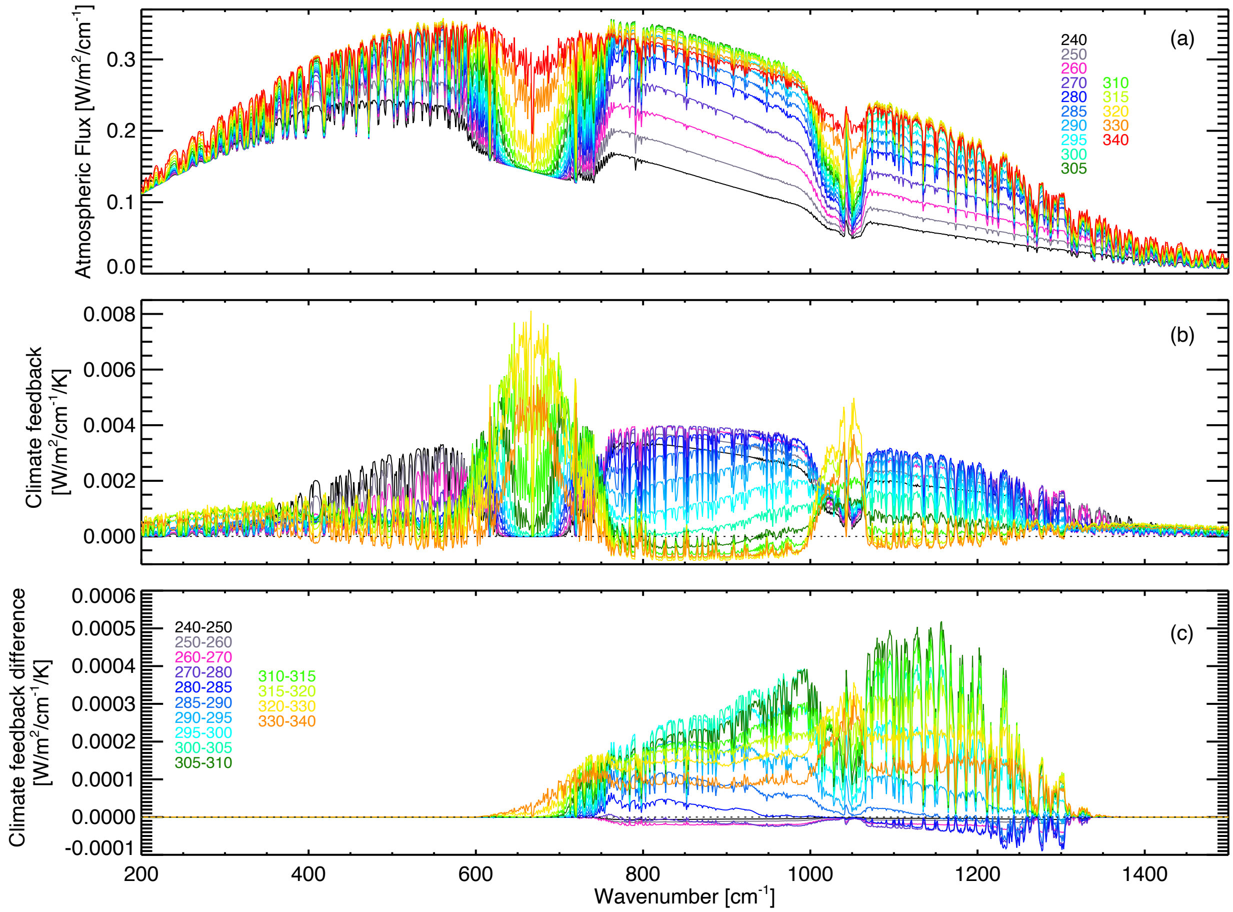

Fig. S4. For various surface temperatures (colored curves) in moist adiabat profiles (as described in text), (a) TOA longwave flux calculated using MT_CKD_4.2_closure, (b) spectral behavior of climate feedback calculated for the temperature ranges denoted on panel (c), and (c) Spectral climate feedback differences between calculations using MT_CKD_4.2_closure and MT_CKD_4.1.1. Comparable to Fig. 23.

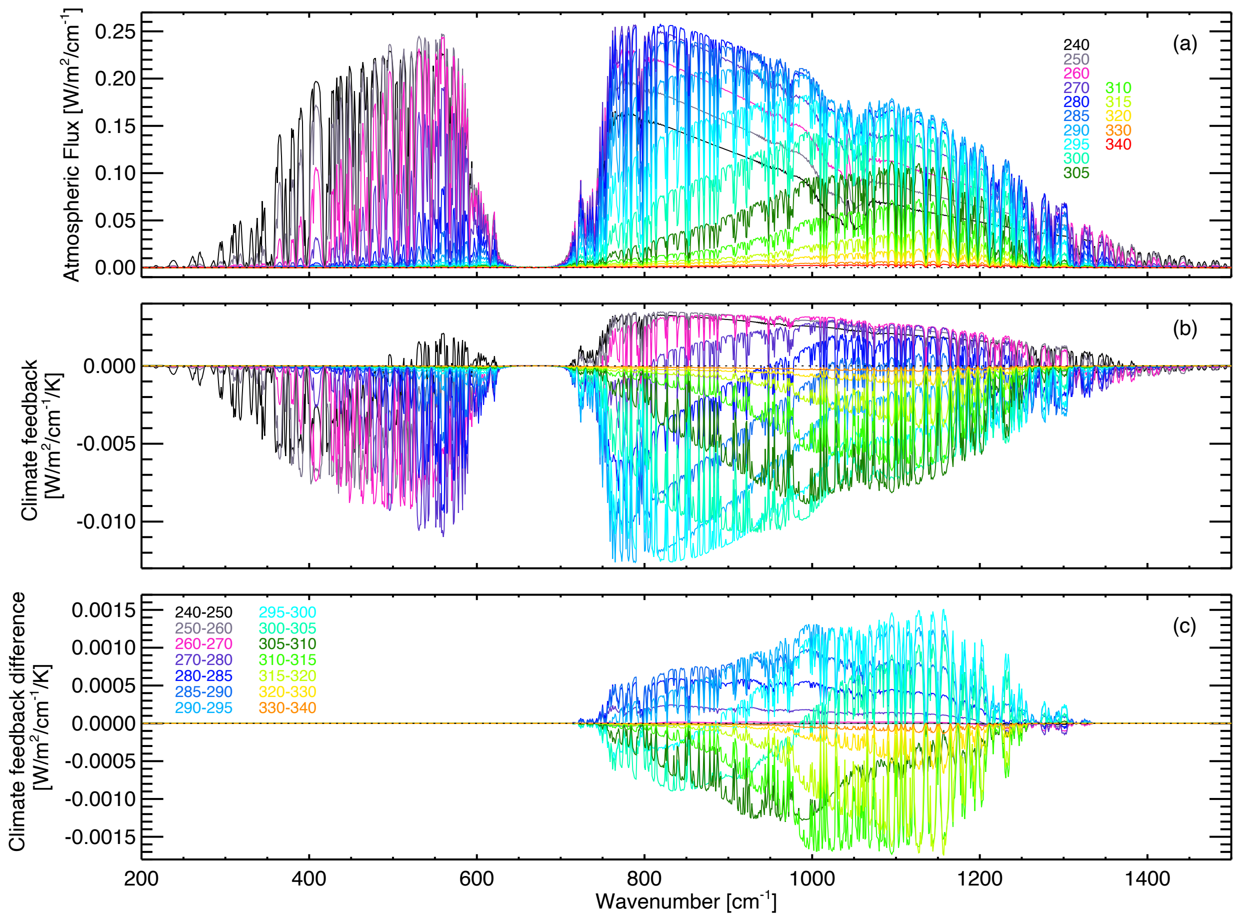

Fig. S5. For various surface temperatures (colored curves) in moist adiabat profiles (as described in text), (a) surface net flux calculated using MT_CKD_4.2, (b) spectral behavior of surface climate feedback calculated for the temperature ranges denoted on panel (c), and (c) spectral surface climate feedback differences between calculations using MT_CKD_4.2 and MT_CKD_4.1.1.

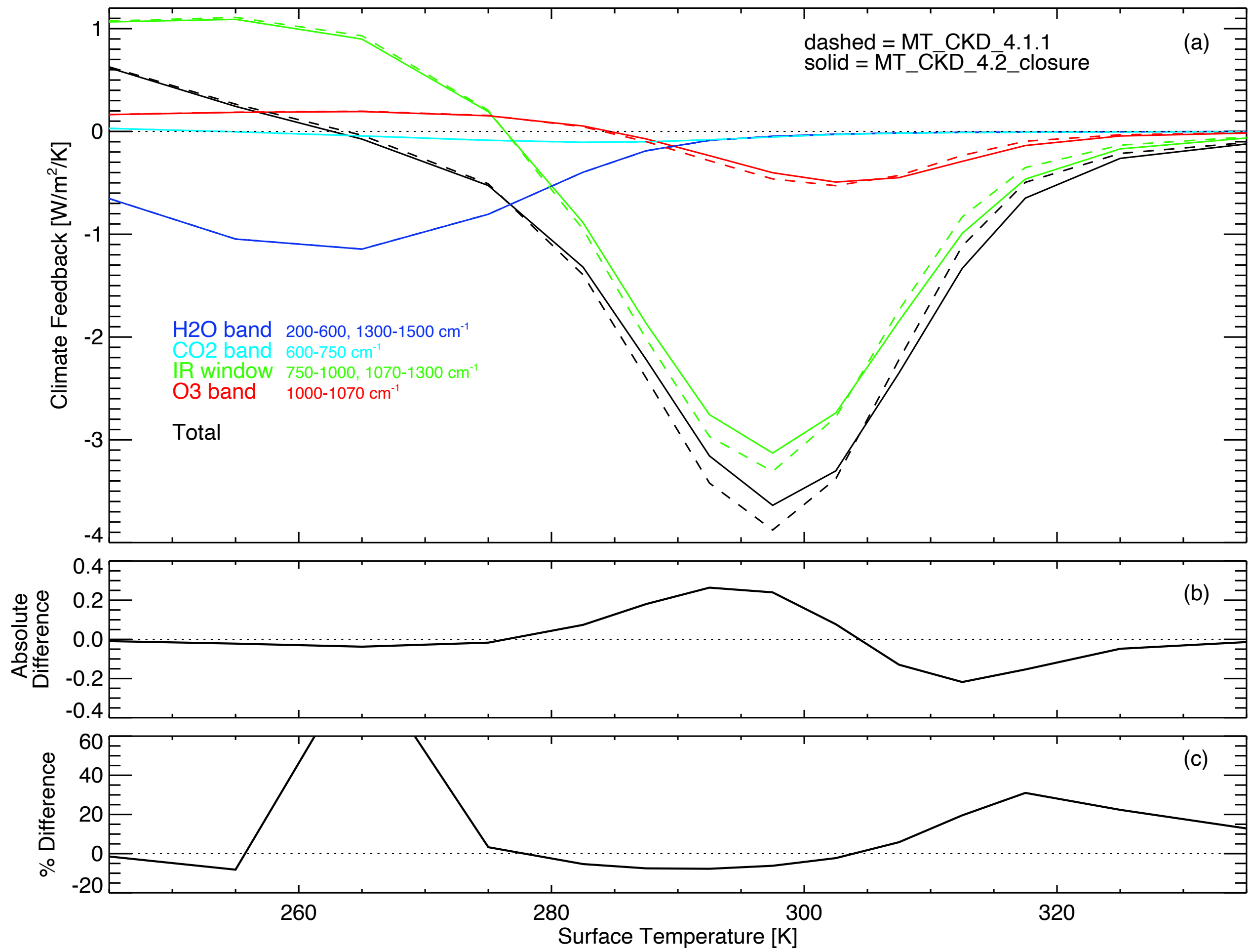

Fig. S6. Comparable to Fig. 24 but for MT_CKD_4.2_closure instead of MT_CKD_4.2.

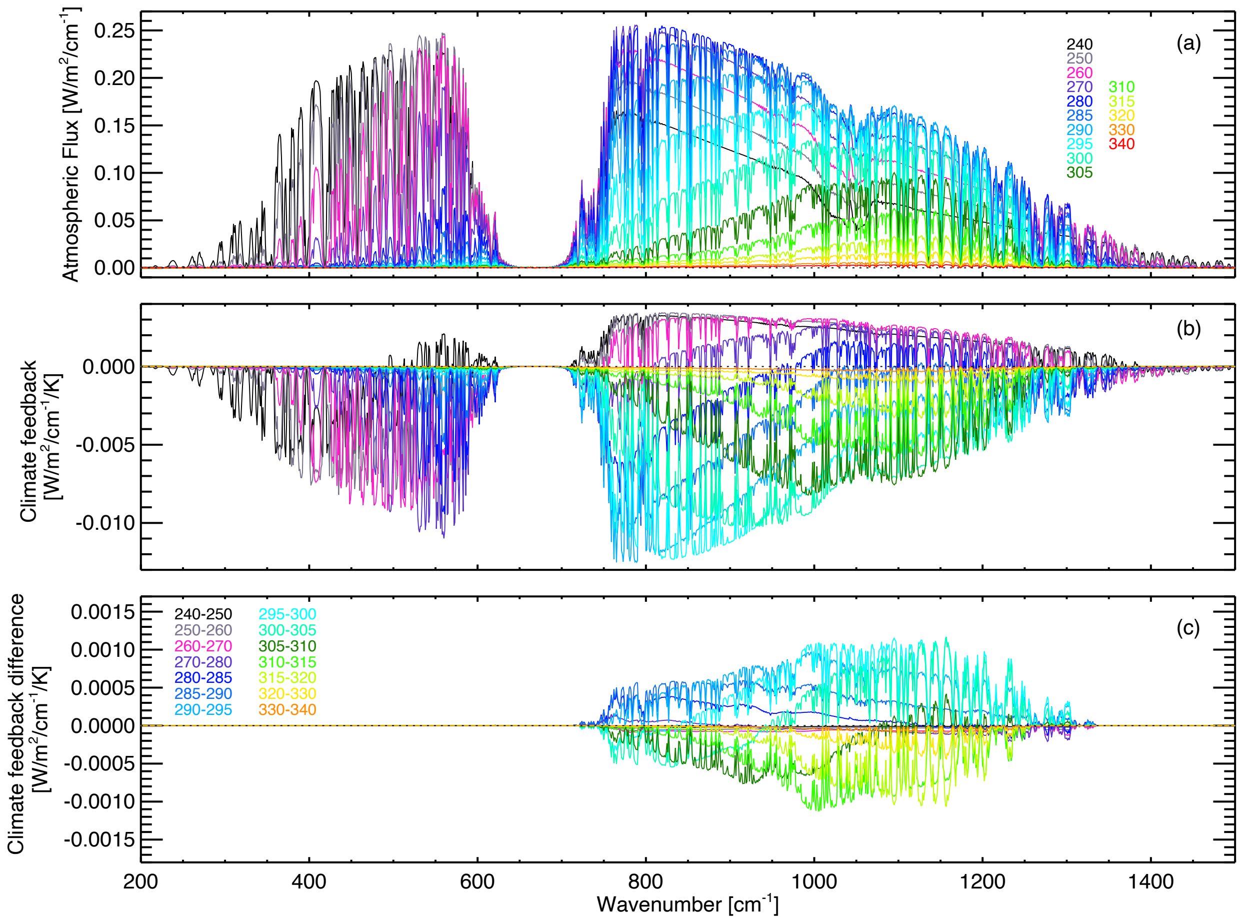

Fig. S7. Comparable to Fig. S5 but for MT_CKD_4.2_closure instead of MT_CKD_4.2.

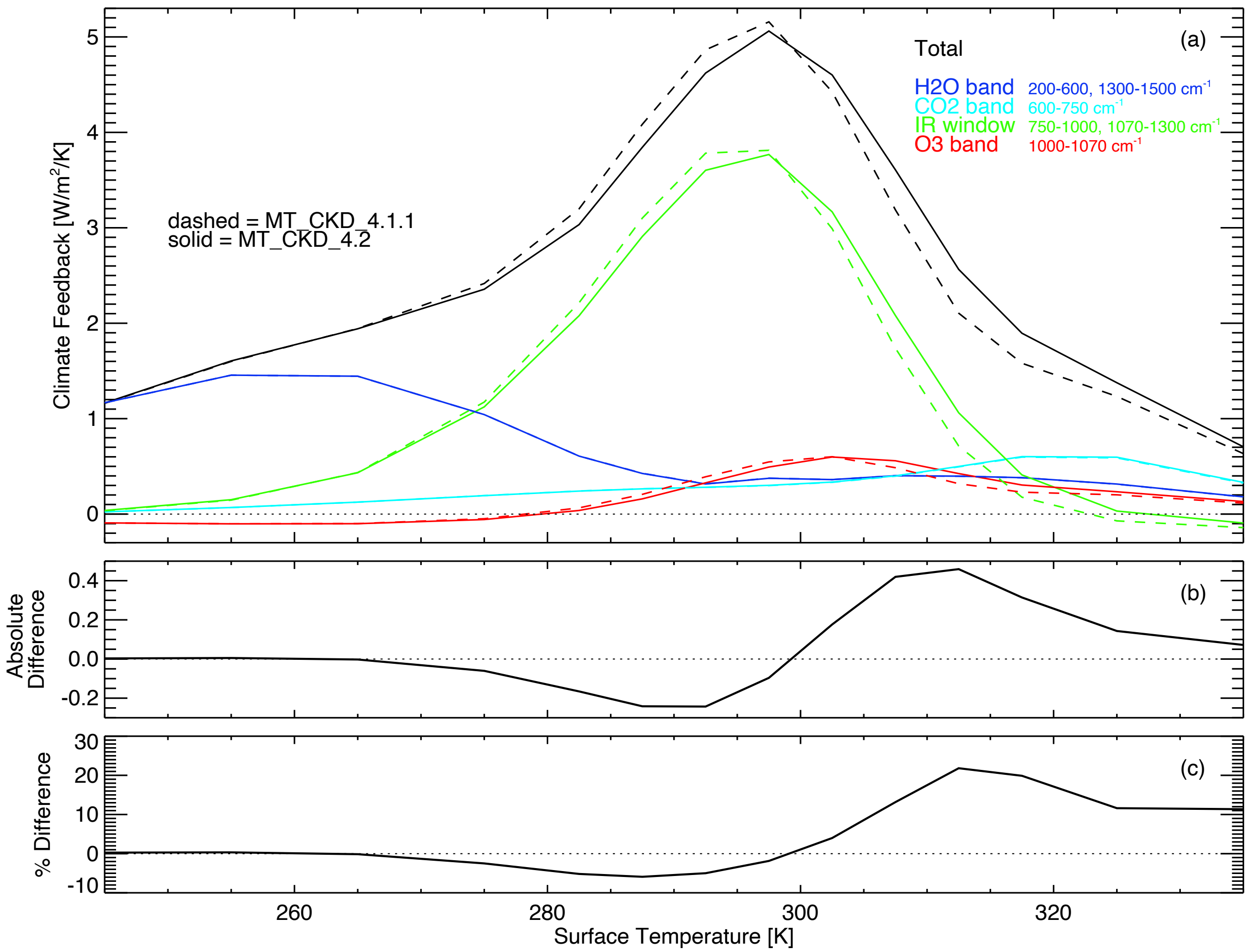

Fig. S8. As a function of surface temperature in moist adiabat profiles (as described in text), (a) atmospheric climate feedback for full longwave region (black), water vapor absorption bands (blue), CO_2 ν_2 band (cyan), infrared window (green), and ozone band (red). Solid curves use revised continuum (MT_CKD_4.2) in the calculations while dashed curves use previous continuum (MT_CKD_4.1.1); (b) for full longwave, atmospheric climate feedback differences between calculations using MT_CKD_4.2 and MT_CKD_4.1.1, and (c) percentage differences in atmospheric climate feedback between calculations using MT_CKD_4.2 and MT_CKD_4.1.1. Atmospheric climate feedback is defined as the difference between climate feedback (i.e. TOA) and surface climate feedback.

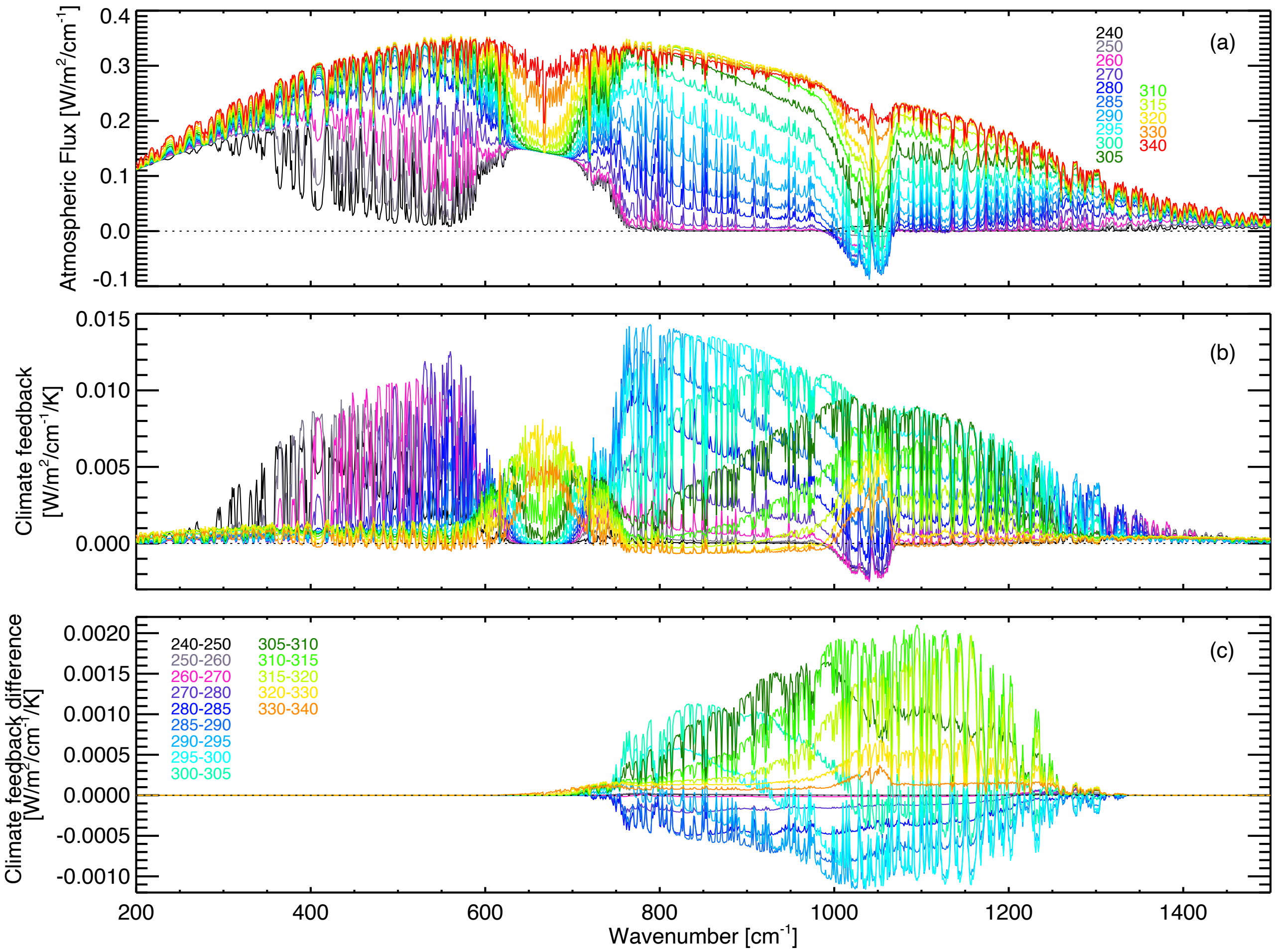

Fig. S9. Similar to Fig. S5 but for the atmospheric flux (a) and atmospheric climate feedback (b,c) instead of the surface flux and feedback, respectively.

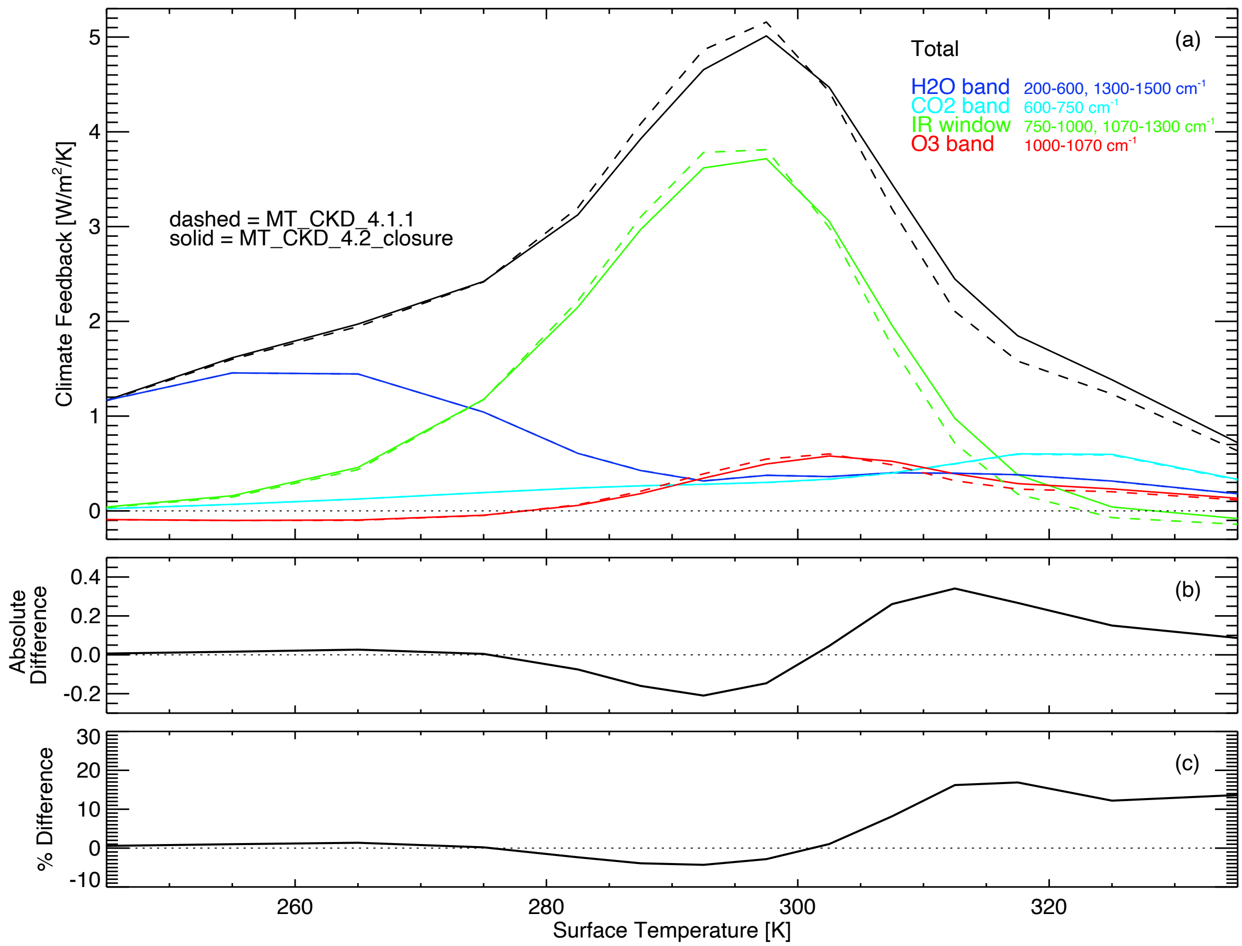

Fig. S10. Similar to Fig. S8 but using MT_CKD_4.2_closure instead of MT_CKD_4.2.

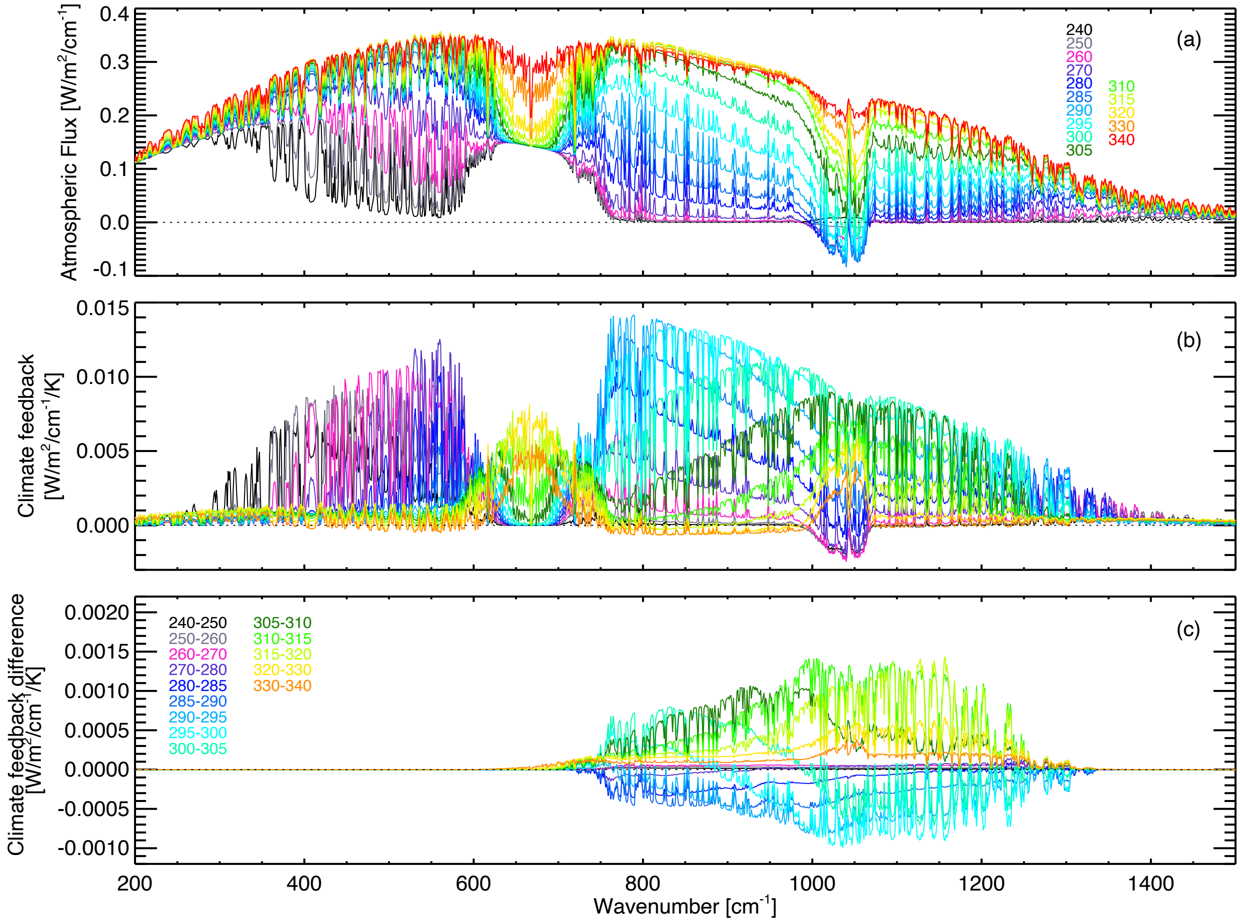

Fig. S11. Similar to Fig. S9 but using MT_CKD_4.2_closure instead of MT_CKD_4.2.

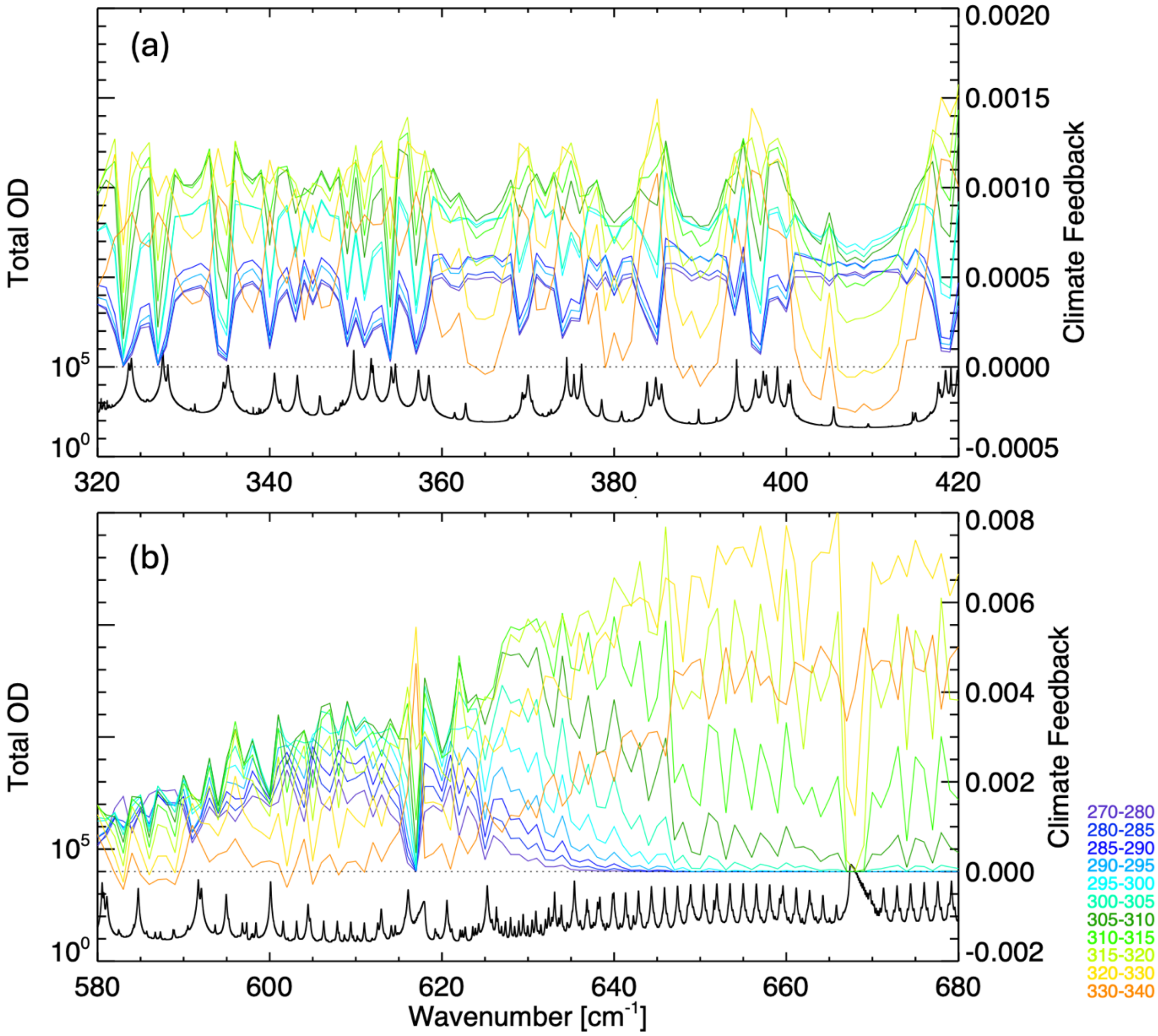

Fig. S12. (a) For 320-420 cm^{-1}, climate feedback ($W/m^2/cm^{-1}/K$, right axis) calculated for LBLRTM calculation using MT_CKD_4.2 for various temperature ranges (colored curves) in moist adiabat profiles (as described in text). Black curves show the optical depths (left axis) for a sample profile; (b) Same as (a) except for the 580-680 cm^{-1} spectral range.

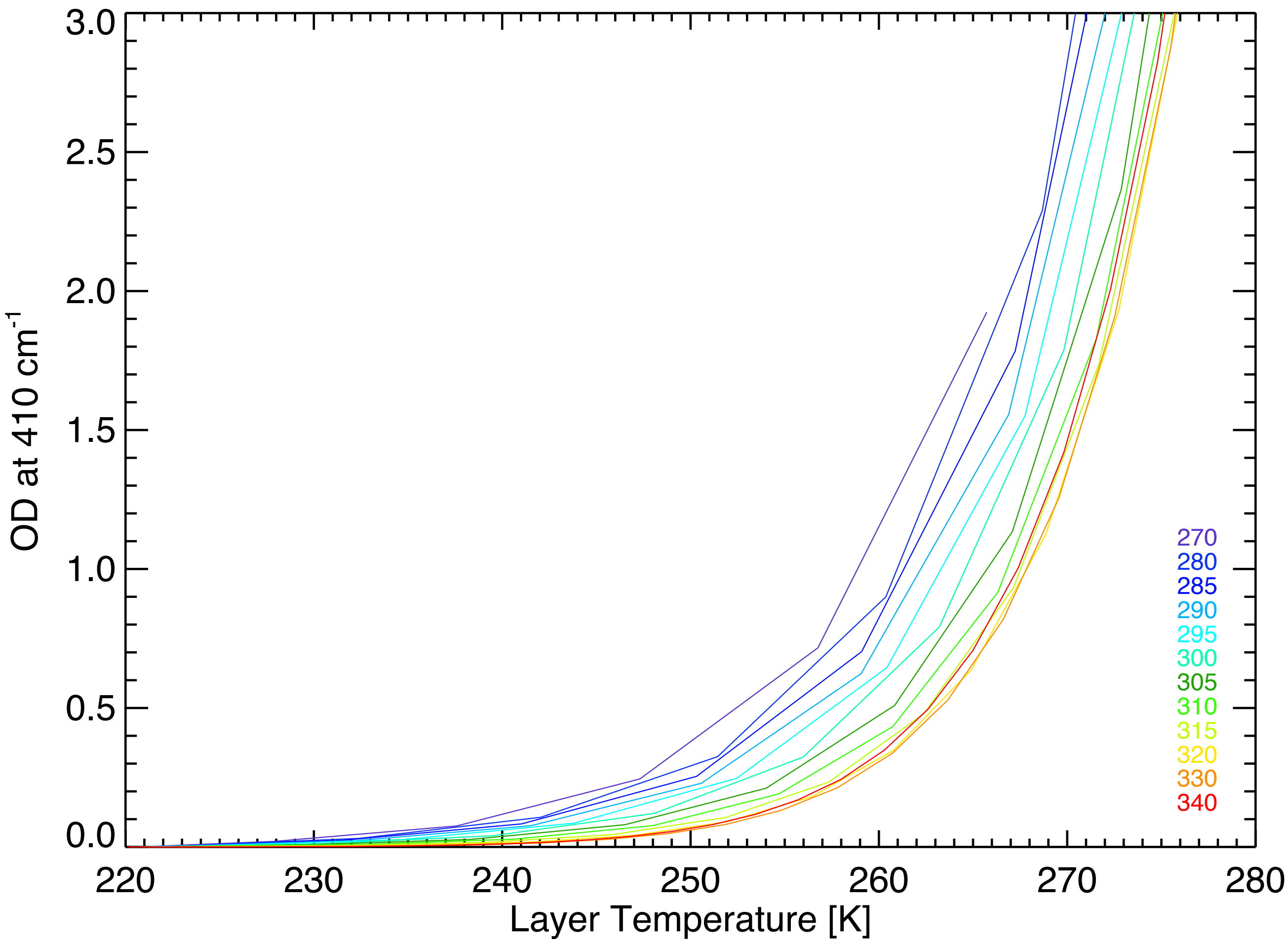

Fig. S13. Cumulative optical depth at 410 cm^{-1} from the tropopause (220K) as a function of atmospheric temperature for various surface temperatures in moist adiabat profiles (as described in text).