Received: Added at production | Revised: Added at production | Accepted: Added at production

DOI: xxx/xxxx

RESEARCH ARTICLE

Dynamic event-triggered fixed-time average consensus for multi-agent systems under switching topologies

Xiaogang Ning[1,2] | Zonggang Li*[1,2] | Yinjuan Chen[1,2] | Yajiang Du[1,2]

[1]School of Mechanical Engineering, Lanzhou Jiaotong University, Lanzhou, China

[2]Robotics Institute, Lanzhou Jiaotong University, Lanzhou, China

Correspondence
*Zonggang Li, School of Mechanical Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China.
Email: lizongg@126.com

Funding information
National Natural Science Foundation of China (61663020);
Gansu Provincial University Industry Support Plan Project (2022CYZC-33);
Open Subject of State Key Laboratory of Structural Analysis for Industrial Equipment of Dalian University of Technology (ZG22119)

Summary

This paper studies the practical fixed-time average consensus problem for general continuous linear multi-agent systems under switching topologies. Firstly, a distributed fixed-time consensus control protocol based on dynamic event-triggered mechanism is designed by using the local information exchange between individual agents. The protocol introduces auxiliary dynamic variables, and the triggering condition changes in real time based on the dynamic variable obtained online, which can significantly reduce the number of triggering events, effectively decrease the energy dissipation of the system and the update frequency of the controller. Then, under the designed control protocol, the sufficient conditions for the multi-agent system to solve the practical fixed-time average consensus problem are given, and it is proved that there is no Zeno behavior in the system. Finally, the simulation results verify the effectiveness of the conclusions.

KEYWORDS:
general linear multi-agent system, dynamic event-triggered control, fixed-time consensus, control protocol, switching topologies

1 | INTRODUCTION

In recent years, the cooperative control of multi-agent systems has attracted great attention from many researchers. It provides important theoretical guidance for the unmanned aerial vehicle(UAV) cluster control,[1] multi-robot formation,[2,3] satellite attitude alignment,[4] control of multiple autonomous underwater vehicles(AUVs),[5] and power management in smart grids.[6,7] It has been widely used in practical systems in military, transportation, and industrial fields.[8] As the basis of cooperative control of multi-agent systems, the consensus problem has always been a research hotspot in the field of control theory. The key to achieving consensus of multi-agent systems is to design appropriate control protocols for the agents by using their own and neighbor information, so that all agents of the system eventually converge to the same state.

In the research of multi-agent systems consensus, fixed-time consensus is an emerging research direction, whose consensus results can be achieved in a specific time and have good steady-state and dynamic characteristics. As an extension of finite-time consensus, the convergence time of fixed-time consensus does not depend on the initial conditions of the system. Therefore, for the actual system with unknown initial state, the control strategy can always ensure the stable convergence of the system in a fixed time, which has more practical significance in engineering applications. The fixed-time consensus control strategy is introduced for the first time in Reference 9, and it is found that the settling time to achieve consensus is independent of the initial state of the system. In Reference 10, a nonlinear control protocol is proposed under the weighted undirected topology, which solves the fixed-time consensus problem of multi-agent systems with nonlinear dynamics and uncertain disturbances. In Reference 11, the

[0]**Abbreviations:** ANA, anti-nuclear antibodies; APC, antigen-presenting cells; IRF, interferon regulatory factor

leader-follower fixed-time consensus for first-order multi-agent systems with unknown inherent nonlinear dynamics is studied, and a distributed control protocol based on local information is proposed to ensure the convergence of the tracking errors in fixed time. In Reference 12, a new class of nonlinear fixed-time consensus protocol is proposed for networked multi-agent systems with directed and intermittent communication, which is extended to solve the fixed-time consensus problem if a common positive dwell time for all active links is strongly connected without the detailed balanced condition. In Reference 13, the fixed-time consensus problem of multi-agent systems under detail-balanced directed graph is studied, and two consensus protocols with linear and nonlinear state measurements are proposed. In Reference 14, a distributed control protocol is designed for the first-order multi-agent systems with discontinuous nonlinear inherent dynamics by using non-smooth analysis and fixed-time stability technology, and the fixed-time consensus of the system under fixed topology and switching topologies is realized. In Reference 15, the fixed-time leader-follower consensus problem for high-order integrator multi-agent systems subject to matched external disturbances is studied, and a fixed-time distributed observer is designed to realize the fixed-time consensus tracking control of the system. In Reference 16, a fully distributed nonsmooth protocol is proposed to achieve fixed-time consensus tracking for a class of first-order multi-agent systems with directed communication topology and unknown disturbances.

It is worth noting that the control protocols proposed in the above References require continuous update of the controller, which often requires the agent to have sufficient computing and energy resources. However, individual agent is usually equipped with limited resources. Therefore, from the perspective of resource constraints, continuous control schemes are impractical, especially for the large multi-agent systems. In order to save a lot of unnecessary communication, computing and energy resources, the fixed-time consensus of multi-agent systems based on event-triggered control is developed. In Reference 17, two distributed event-triggered fixed-time consensus protocols are proposed for leader-follower multi-agent systems with nonlinear dynamics and uncertain disturbances, which greatly reduce the energy consumption and update frequency of the controller. In Reference 18, two event-triggered fixed-time consensus controllers are designed to solve the fixed-time consensus problem of uncertain nonlinear multi-agent systems under continuous communication and intermittent communication. In Reference 19, a fixed-time consensus protocol based on event-triggered control is proposed for multi-agent systems with input delay and uncertain disturbances, which avoids continuous communication in controller update and triggering state monitoring. In Reference 20, the fixed-time consensus problem of second-order multi-agent systems with uncertain bounded disturbances is studied, and an event-triggered control protocol which is completely distributed and does not require continuous communication is designed. In Reference 21, two fixed-time leader-follower consensus control protocols are proposed for nonlinear networked multi-agent systems based on event/self-triggered mechanism, and the self-triggered control strategy avoids continuous triggering condition monitoring of the system. In Reference 22, the fixed-time consensus problem of linear multi-agent systems with input delay is studied, and two distributed event-triggered control protocols are designed to achieve fixed-time leaderless consensus and leader-follower consensus. In Reference 23, an event-triggered control protocol is proposed for nonlinear multi-agent systems with input delay, external disturbances and switching topologies to achieve fixed-time average consensus. In addition, an improved control protocol without continuous communication is also given.

At present, some researchers have introduced the dynamic event-triggered mechanism into the fixed-time consensus control of multi-agent systems. The dynamic event-triggered mechanism is not only related to the state of the system, but also considers an additional auxiliary dynamic variable, which can be obtained online and adjusted by design parameters. Therefore, the triggering condition of the system changes in real time, which effectively reduces the number of triggering events. In Reference 24, a dynamic event-triggered control protocol based on internal dynamic variable is proposed to solve the practical fixed-time consensus problem of first-order multi-agent systems with nonlinear dynamics. In Reference 25, the fixed-time cooperative tracking problem under dynamic event-triggered control is studied for first-order multi-agent systems with input delay and unknown disturbances. In Reference 26, a control protocol based on dynamic event-triggered mechanism is proposed for first-order nonlinear multi-agent systems with disturbances, and the practical fixed-time average consensus of the system under switching topologies is realized.

It should be pointed out that the event-triggered mechanism in the above References 17-23 does not introduce auxiliary dynamic variable and belongs to static event-triggered control. Therefore, the number of triggering events is more than that of dynamic event-triggered control, resulting in unnecessary waste of resources. In References 24-26, the dynamic event-triggered mechanism is introduced into the fixed-time consensus control of multi-agent systems, but the research is carried out for first-order multi-agent systems, which is not involved in general linear multi-agent systems. In References 17-22,24,25, the communication topologies are all fixed, and the more realistic switching topologies are not considered. So the communication conditions are not general.

Inspired by these works of the above References, this paper innovatively designs a control protocol based on the dynamic event-triggered mechanism, which solves the practical fixed-time average consensus problem for the general continuous linear multi-agent system under switching topologies, and proves that the system does not have Zeno behavior. As far as we know, the practical fixed-time average consensus problem of general linear multi-agent systems based on dynamic event-triggered mechanism under switching topologies is still open. The main contributions of this paper involve three aspects.

1. Compared with the static event-triggered mechanism,[17,18,19,20,21,22,23] this paper proposes a dynamic event-triggered control scheme based on auxiliary dynamic variables, which can effectively reduce the number of triggering events and save system resources.
2. Different from the analysis of fixed-time consensus problem for fixed topology,[17,18,19,20,21,22,24,25] this paper solves the practical fixed-time average consensus problem of general linear multi-agent systems under a more general switching topologies.
3. Compared with the research on first-order systems,[24,25,26] this paper breaks the limitations of first-order systems and extends the dynamic event-triggered mechanism to general linear multi-agent systems, which poses a greater challenge to the design of fixed-time consensus control protocol.

The rest of this paper is organized as follows. In Section 2, preparatory knowledge and problem description are given. In Section 3, the main results including the design of distributed dynamic event-triggered control protocol and the analysis of fixed-time average consensus are given. In Section 4, an example simulation is provided to verify the theoretical results, and the parameter analysis and comparative experiments are carried out. Finally, the conclusions are drawn in Section 5.

2 | PREPARATORY KNOWLEDGE AND PROBLEM DESCRIPTION

2.1 | Graph theory and notations

For a multi-agent system composed of N agents, the communication network between agents can be represented by an undirected topology graph, denoted by $\mathcal{G}^{\sigma(t)} = (\mathcal{V}, \mathcal{E}^{\sigma(t)}, \mathcal{A}^{\sigma(t)})$, and the N agents can be regarded as N nodes. Among them, $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ is the node set, $\mathcal{E}^{\sigma(t)} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set, $\mathcal{A}^{\sigma(t)} = [a_{ij}(t)] \in R^{N\times N}$ is the adjacency matrix, $\sigma(t)$ is the piecewise constant switching signal. Define the switching signal $\sigma(t) : [0, +\infty) \to \mathcal{P} = \{1, \cdots, p\}, p \in \mathbb{N}^+$, $\mathcal{P}$ denotes the index set of all the switching topology graphs. Define the diagonal element $a_{ii}(t) = 0$ of the adjacency matrix $\mathcal{A}^{\sigma(t)}$, and when $(v_i, v_j) \in \mathcal{E}^{\sigma(t)}$, there is $a_{ij}(t) = 1$, otherwise $a_{ij}(t) = 0$. The edge (v_i, v_j) here indicates that agent i can obtain information from agent j. If $(v_j, v_i) \in \mathcal{E}^{\sigma(t)} = (v_i, v_j) \in \mathcal{E}^{\sigma(t)}$, then $\mathcal{G}^{\sigma(t)}$ is an undirected graph. An undirected graph $\mathcal{G}^{\sigma(t)}$ is said to be connected if there exists a path between every pair of distinct nodes. The Laplacian matrix $\mathcal{L}^{\sigma(t)} = [l_{ij}(t)] \in R^{N\times N}$ of $\mathcal{G}^{\sigma(t)}$, where $l_{ii}(t) = \sum_{j=1}^{N} a_{ij}(t)$ and $l_{ij}(t) = -a_{ij}(t)$. The neighbor set of agent i is expressed as $\mathcal{N}_i^{\sigma(t)} = \{v_j \in \mathcal{V} | (v_i, v_j) \in \mathcal{E}^{\sigma(t)}\}$.

In this paper, $R^{n\times m}$ denotes a set of $n \times m$-dimensional real matrices, R^n denotes a set of n-dimensional real vectors, I_n denotes the n-dimensional identity matrix, $\mathbf{1}_n(\mathbf{0}_n)$ denotes the n-dimensional column vector with all elements being 1(0), $diag\{\cdot\}$ represents the diagonal matrix, $col(\cdot)$ represents the column vector, $\otimes$ denotes the Kronecker product of matrices, $\|\cdot\|$ denotes the Euclidean norm of a vector or the compatible matrix norm. For any vector $x = col(x_1, x_2, \cdots, x_N) \in R^N$, define $sign(x) = col(sign(x_1), sign(x_2), \cdots, sign(x_N))$ and $sig^{\mu}(x) = col(sig^{\mu}(x_1), sig^{\mu}(x_2), \cdots, sig^{\mu}(x_N))$, where $sig^{\mu}(x_i) = |x_i|^{\mu} sign(x_i), i = 1, 2, \cdots, N,\ \mu > 0$.

2.2 | Definitions and lemmas

Consider the following dynamic system

$$\begin{cases} \dot{x}(t) = f(t, x(t)) \\ x(0) = x_0 \end{cases} \tag{1}$$

where $x(t) \in R^n$ denotes the state vector, $f(t, x(t)) : R^+ \times R^n \to R^n$ is a smooth function. Suppose that the origin is the equilibrium point of the system (1), namely $f(t, \mathbf{0}) = \mathbf{0}$.

Definition 1 ([9,27]). If the origin of the system (1) is asymptotically stable and there exists a settling time $T(x_0) > 0$ such that any solution $x(t, x_0)$ of the system can converge to the equilibrium point at $T(x_0)$, then the origin of the system (1) is globally finite-time stable. Furthermore, if there exists $T_{\max} > 0$ such that the stabilization time $T \leq T_{\max}$ is satisfied for any initial state of the system, then the origin of the system (1) is fixed-time stable. In addition, if any solution of the system can only converge to a sufficiently small neighborhood of the equilibrium point, the origin of the system (1) is practical fixed-time stable.

Lemma 1 ([26,28]). If there exists a Lyapunov function $V(x(t)) : R^n \to R^+$ such that

$$\dot{V}(x(t)) \leq -aV^p(x(t)) - bV^q(x(t)) + \vartheta \tag{2}$$

holds, where $a, b, \vartheta > 0$, $p \in (0,1), q \in (1, +\infty)$, then the origin of the system (1) is practical fixed-time stable, and the settling time T satisfies $T \leq T_{\max} = \frac{1}{a\kappa(1-p)} + \frac{1}{b\kappa(q-1)}$, where κ is a scalar and $0 < \kappa < 1$. In addition, the residual error set of the solution of the system (1) is $\{\lim_{t\to T} x(t) | V(x(t)) \leq \min\{a^{-\frac{1}{p}}(\frac{\vartheta}{1-\kappa})^{\frac{1}{p}}, b^{-\frac{1}{q}}(\frac{\vartheta}{1-\kappa})^{\frac{1}{q}}\}\}$.

Lemma 2 ([29,30]). Let $\mathcal{G}$ be a connected undirected graph of order N, and $\mathcal{L}$ is the corresponding Laplacian matrix. If the eigenvalues of $\mathcal{L}$ are expressed as $\lambda_1(\mathcal{L}), \lambda_2(\mathcal{L}), \cdots, \lambda_N(\mathcal{L})$, then there exists $0 = \lambda_1(\mathcal{L}) < \lambda_2(\mathcal{L}) \leq \cdots \leq \lambda_N(\mathcal{L})$.

Lemma 3 ([19,31]). For $x_i \in R, i = 1, 2, \cdots, N$, when $0 < p \leq 1, 1 < q < \infty$, there exist

$$\begin{cases} N^{1-p}\Big(\sum_{i=1}^{N} |x_i|\Big)^p \geq \sum_{i=1}^{N} |x_i|^p \geq \Big(\sum_{i=1}^{N} |x_i|\Big)^p \\ N^{1-q}\Big(\sum_{i=1}^{N} |x_i|\Big)^q \leq \sum_{i=1}^{N} |x_i|^q \leq \Big(\sum_{i=1}^{N} |x_i|\Big)^q \end{cases}$$

2.3 | Problem description

This paper considers a multi-agent system consisting of N agents. The dynamic model of agent i is described as

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \tag{3}$$

where $i = 1, 2, \cdots, N$, $x_i(t) \in R^n$ represents the state vector, $u_i(t) \in R^m$ represents the control input, $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are constant matrices.

The goal of this paper is to design a distributed control protocol based on dynamic event-triggered mechanism for the general linear multi-agent system (1), so that each agent can achieve average consensus in a fixed time by using only local information and without continuous controller update. To this end, before designing the control protocol, the relevant assumptions and definitions required by the system are given.

Assumption 1. The matrix B is row full rank, that is, there exists a matrix $K \in R^{m\times n}$ such that $BK = I_n$.

Assumption 2. The undirected communication topology graph $\mathcal{G}^{\sigma(t)}$ is connected at any time t.

Assumption 3. For the switching signal $\sigma(t) : [0, +\infty) \to \mathcal{P} = \{1, \cdots, p\}$, the set $\mathcal{P}$ is finite, and $t_0 < t_1 < \cdots < t_s < t_{s+1} < \cdots$ is the corresponding switching instant. Within the adjacent switching instant interval $[t_s, t_{s+1})$, the communication topology is fixed, and there is $\tau < t_{s+1} - t_s < \mathcal{T}$, where $s = 0, 1, \cdots$, the residence time τ and switching period $\mathcal{T}$ are positive constants.

Definition 2 ([23]). For the multi-agent system (3), if there exist a time T and a sufficiently small positive number ϵ such that any agent i satisfies $\lim_{t\to T} \|x_i(t) - \frac{1}{N}\sum_{j=1}^{N} x_j(t)\| \leq \epsilon$, and when $t \geq T$, $\|x_i(t) - \frac{1}{N}\sum_{j=1}^{N} x_j(t)\| \leq \epsilon$ still holds, and the system also has $T_{\max} > 0$ such that $T \leq T_{\max}$ exists under any initial conditions, then the multi-agent system (3) achieves the practical fixed-time average consensus.

3 | MAIN RESULTS

3.1 | Design of distributed dynamic event-triggered control protocol

For the multi-agent system (3), based on the event-triggered control method and by using the local information exchange between neighbors, a distributed fixed-time consensus controller for agent i is designed as

$$u_i(t) = -\gamma_1 B^T P\xi_i(t_k^i) - \gamma_2 K sign(P\xi_i(t_k^i)) - \gamma_3 K sig^{\mu}(P\xi_i(t_k^i)), \tag{4}$$

where $t \in [t_k^i, t_{k+1}^i)$, $\gamma_1, \gamma_2, \gamma_3 > 0$ are the design parameters, $\mu > 1$ is the ratio of two positive odd numbers, $P \in R^{n\times n}$ is a positive definite matrix to be determined, the matrix $K \in R^{m\times n}$ can be obtained by Assumption 1, t_k^i is the kth triggering instant of agent i, and it is considered that the control input of agent i is unchanged before the next triggering instant t_{k+1}^i of the latest triggering instant t_k^i. In addition, $\xi_i(t) \in R^n$ represents the relative state measurement, and $\xi_i(t) = \sum_{j=1}^{N} a_{ij}(t)(x_i(t) - x_j(t))$ is defined.

Remark 1. For the distributed event-triggered controller under the switching topologies, the switching instant of the system topology graph is also considered as the event-triggered instant of each agent, because the change of the communication topology will generate new local information. That is,

$$t_k^i = \begin{cases} t_s, & \text{topology switching instant} \\ t_k^i. & \text{event triggering instant} \end{cases} \tag{5}$$

Therefore, agent i only performs control update at its own event-triggered instant and the topology switching instant.

The measurement error of agent i is defined as

$$\begin{aligned} e_i(t) =&\gamma_1 B^T P\xi_i(t_k^i) + \gamma_2 K sign(P\xi_i(t_k^i)) + \gamma_3 K sig^{\mu}(P\xi_i(t_k^i)) - \\ &\gamma_1 B^T P\xi_i(t) - \gamma_2 K sign(P\xi_i(t)) - \gamma_3 K sig^{\mu}(P\xi_i(t)). \end{aligned} \tag{6}$$

Then, the event-triggered function of agent i is designed as

$$g_i(t) = \|e_i(t)\|^2 - \eta_i^2 \xi_i^T(t) PBB^T P\xi_i(t), \tag{7}$$

where $\eta_i > 0$ is the design parameter.

Inspired by References 22 and 32, this paper considers the use of dynamic event-triggered mechanism. Therefore, design the auxiliary dynamic variable $\phi_i(t)$ for agent i as follows:

$$\dot{\phi}_i(t) = -2\alpha\phi_i^{\frac{1}{2}}(t) - 2\beta\phi_i^{\frac{\mu+1}{2}}(t) - \xi_i^T(t)PBe_i(t) + \tau, \tag{8}$$

where $\alpha > 0, \beta > 0, \tau > 0, \phi_i(0) > 0$ are the design parameters that can be selected as required.

The dynamic event-triggered condition of agent i is constructed as

$$t_{k+1}^i = \inf\{t > t_k^i | g_i(t) \geq f(\phi_i(t))\}, \quad k = 0, 1, \cdots, \tag{9}$$

where $f(\phi_i(t)) = \eta_i\alpha\phi_i^{\frac{1}{2}}(t) + \eta_i\beta\phi_i^{\frac{\mu+1}{2}}(t)$.

Remark 2. The event-triggered condition (9) is distributed. According to (9), for agent i, $g_i(t) \leq f(\phi_i(t))$ is totally satisfied at any $t \in [t_k^i, t_{k+1}^i)$, and $\|e_i(t)\|^2 \leq \eta_i^2\xi_i^T(t)PBB^TP\xi_i(t) + \eta_i\alpha\phi_i^{\frac{1}{2}}(t) + \eta_i\beta\phi_i^{\frac{\mu+1}{2}}(t)$ always holds at this time. In addition, based on (8), $\phi_i(t) > 0$ can be obtained for any $t \geq 0$ by designing appropriate parameters.

3.2 | Analysis of fixed-time average consensus

The average consensus error of agent i is defined as

$$\delta_i(t) = x_i(t) - \frac{1}{N}\sum_{j=1}^{N} x_j(t). \tag{10}$$

Then there is

$$\delta(t) = (M \otimes I_n)x(t), \tag{11}$$

where $M = I_N - \frac{1}{N}1_N 1_N^T, \delta(t) = col(\delta_1(t), \delta_2(t), \cdots, \delta_N(t)), x(t) = col(x_1(t), x_2(t), \cdots, x_N(t))$.

The derivative of (11) with respect to time t can be expressed as

$$\dot{\delta}(t) = (M \otimes I_n)\dot{x}(t). \tag{12}$$

According to (6), the distributed controller (4) can be rewritten as

$$u_i(t) = -e_i(t) - \gamma_1 B^T P\xi_i(t) - \gamma_2 K sign(P\xi_i(t)) - \gamma_3 K sig^{\mu}(P\xi_i(t)). \tag{13}$$

Substituting (13) into (3) yields that

$$\dot{x}_i(t) = Ax_i(t) - Be_i(t) - \gamma_1 BB^T P\xi_i(t) - \gamma_2 sign(P\xi_i(t)) - \gamma_3 sig^{\mu}(P\xi_i(t)). \tag{14}$$

The (14) is rewritten into a compact supervector form as follows:

$$\begin{aligned}\dot{x}(t) =&(I_N \otimes A)x(t) - (I_N \otimes B)e(t) - \gamma_1(\mathcal{L}^{\sigma(t)} \otimes BB^T P)x(t) - \\ &\gamma_2 sign((\mathcal{L}^{\sigma(t)} \otimes P)x(t)) - \gamma_3 sig^{\mu}((\mathcal{L}^{\sigma(t)} \otimes P)x(t)),\end{aligned} \tag{15}$$

where $e(t) = col(e_1(t), e_2(t), \cdots, e_N(t))$.

Substituting (15) into (12) yields that

$$\begin{aligned}\dot{\delta}(t) =&(I_N \otimes A)\delta(t) - (M \otimes B)e(t) - \gamma_1(M\mathcal{L}^{\sigma(t)} \otimes BB^T P)\delta(t) - \\ &\gamma_2(M \otimes I_n)sign((\mathcal{L}^{\sigma(t)} \otimes P)\delta(t)) - \gamma_3(M \otimes I_n)sig^{\mu}((\mathcal{L}^{\sigma(t)} \otimes P)\delta(t)).\end{aligned} \tag{16}$$

Thus, the practical fixed-time average consensus problem of multi-agent system (3) is transformed into the fixed-time stable convergence problem of system (16).

Theorem 1. For the multi-agent system (3) satisfying Assumptions 1-3, the individual controller and event-triggered condition are given by (4) and (9), respectively. If there exists a positive definite matrix P such that the following linear matrix inequality(LMI)

$$PA + A^T P - 2PBB^T P < -\gamma P \tag{17}$$

holds, and the conditions

$$\gamma_1 \geq \frac{1}{\lambda_{2-\min}} + 2\eta_{\max}, \tag{18}$$

$$\gamma_2 \geq \frac{\alpha}{(2\lambda_{2-\min}\lambda_{\min}(P))^{\frac{1}{2}}}, \tag{19}$$

$$\gamma_3 \geq \frac{\beta}{n^{\frac{1-\mu}{2}}(2\lambda_{2-\min}\lambda_{\min}(P))^{\frac{\mu+1}{2}}}, \tag{20}$$

$$\gamma \geq 0 \tag{21}$$

hold at the same time, where $\lambda_{2-\min} = \min\{\lambda_2(\mathcal{L}^{\sigma(t)})\}$, then the system (3) can achieve the practical fixed-time average consensus.

Proof. Consider the Lyapunov candidate function as follows:

$$V(t) = \frac{1}{2}\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes P)\delta(t) + \sum_{i=1}^{N} \phi_i(t). \tag{22}$$

Let $V_1(t) = \frac{1}{2}\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes P)\delta(t),\ V_2(t) = \sum_{i=1}^{N} \phi_i(t)$, then

$$V(t) = V_1(t) + V_2(t).$$

The derivative of $V_1(t)$ with respect to time t can be expressed as

$$\begin{aligned}\dot{V}_1(t) =&\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes P)\dot{\delta}(t) \\ =&\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes P)\big((I_N \otimes A)\delta(t) - \gamma_1(M\mathcal{L}^{\sigma(t)} \otimes BB^T P)\delta(t) - (M \otimes B)e(t) - \\ &\gamma_2(M \otimes I_n)sign((\mathcal{L}^{\sigma(t)} \otimes P)\delta(t)) - \gamma_3(M \otimes I_n)sig^{\mu}((\mathcal{L}^{\sigma(t)} \otimes P)\delta(t))\big) \\ =&\delta^T(t)\Big(\mathcal{L}^{\sigma(t)} \otimes \frac{PA + A^T P}{2} - \gamma_1(\mathcal{L}^{\sigma(t)}\mathcal{L}^{\sigma(t)} \otimes PBB^T P)\Big)\delta(t) - \delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes PB)e(t) - \\ &\gamma_2\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes P)sign((\mathcal{L}^{\sigma(t)} \otimes P)\delta(t)) - \gamma_3\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes P)sig^{\mu}((\mathcal{L}^{\sigma(t)} \otimes P)\delta(t)).\end{aligned} \tag{23}$$

The derivative of $V_2(t)$ with respect to time t can be expressed as

$$\dot{V}_2(t) = \sum_{i=1}^{N} \dot{\phi}_i(t) = -2\alpha \sum_{i=1}^{N} \phi_i^{\frac{1}{2}}(t) - 2\beta \sum_{i=1}^{N} \phi_i^{\frac{\mu+1}{2}}(t) - \sum_{i=1}^{N} \xi_i^T(t)PBe_i(t) + N\tau. \tag{24}$$

According to (23) and (24), we can get

$$\begin{aligned}\dot{V}(t) =&\delta^T(t)\Big(\mathcal{L}^{\sigma(t)} \otimes \frac{PA + A^T P}{2} - \gamma_1(\mathcal{L}^{\sigma(t)}\mathcal{L}^{\sigma(t)} \otimes PBB^T P)\Big)\delta(t)- \\ &\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes PB)e(t) - \sum_{i=1}^{N} \xi_i^T(t)PBe_i(t) - 2\alpha \sum_{i=1}^{N} \phi_i^{\frac{1}{2}}(t) - 2\beta \sum_{i=1}^{N} \phi_i^{\frac{\mu+1}{2}}(t)- \\ &\gamma_2\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes P)sign((\mathcal{L}^{\sigma(t)} \otimes P)\delta(t)) - \gamma_3\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes P)sig^\mu((\mathcal{L}^{\sigma(t)} \otimes P)\delta(t)) + N\tau.\end{aligned} \tag{25}$$

By using Young's inequality, the second and third terms on the right side of the equal sign of (25) can be transformed into

$$\begin{aligned}&-\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes PB)e(t) - \sum_{i=1}^{N} \xi_i^T(t)PBe_i(t) \\ &= -2\sum_{i=1}^{N} \xi_i^T(t)PBe_i(t) \le \sum_{i=1}^{N} \eta_i\xi_i^T(t)PBB^T P\xi_i(t) + \sum_{i=1}^{N} \frac{e_i^T(t)e_i(t)}{\eta_i}.\end{aligned} \tag{26}$$

Combining with (26), and according to the event-triggered condition (9), the second, third, fourth, and fifth items on the right side of the equal sign of (25) can be transformed into

$$\begin{aligned}&-\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes PB)e(t) - \sum_{i=1}^{N} \xi_i^T(t)PBe_i(t) - 2\alpha \sum_{i=1}^{N} \phi_i^{\frac{1}{2}}(t) - 2\beta \sum_{i=1}^{N} \phi_i^{\frac{\mu+1}{2}}(t) \\ &\le 2\sum_{i=1}^{N} \eta_i\xi_i^T(t)PBB^T P\xi_i(t) - \alpha \sum_{i=1}^{N} \phi_i^{\frac{1}{2}}(t) - \beta \sum_{i=1}^{N} \phi_i^{\frac{\mu+1}{2}}(t) \\ &\le 2\eta_{\max}\delta^T(t)(\mathcal{L}^{\sigma(t)}\mathcal{L}^{\sigma(t)} \otimes PBB^T P)\delta(t) - \alpha\Big(\sum_{i=1}^{N} \phi_i(t)\Big)^{\frac{1}{2}} - \beta N^{\frac{1-\mu}{2}}\Big(\sum_{i=1}^{N} \phi_i(t)\Big)^{\frac{\mu+1}{2}},\end{aligned} \tag{27}$$

where $\eta_{\max} = \max\{\eta_1, \eta_2, \cdots, \eta_N\}$.

Let $(\mathcal{L}^{\sigma(t)} \otimes P)\delta(t) = z(t) = col(z_1(t), z_2(t), \cdots, z_{Nn}(t))$, then according to Lemma 3, the sixth term on the right side of the equal sign of (25) can be written as

$$\begin{aligned}&-\gamma_2\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes P)sign((\mathcal{L}^{\sigma(t)} \otimes P)\delta(t)) \\ &= -\gamma_2 \sum_{i=1}^{Nn} |z_i(t)| = -\gamma_2 \sum_{i=1}^{Nn} (z_i^2(t))^{\frac{1}{2}} \\ &\le -\gamma_2\Big(\sum_{i=1}^{Nn} z_i^2(t)\Big)^{\frac{1}{2}} = -\gamma_2\Big(\delta^T(t)(\mathcal{L}^{\sigma(t)}\mathcal{L}^{\sigma(t)} \otimes PP)\delta(t)\Big)^{\frac{1}{2}}.\end{aligned} \tag{28}$$

Similarly, the seventh term on the right side of the equal sign of (25) can be written as

$$\begin{aligned}&-\gamma_3\delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes P)sig^\mu((\mathcal{L}^{\sigma(t)} \otimes P)\delta(t)) \\ &= -\gamma_3 \sum_{i=1}^{Nn} |z_i(t)|^{\mu+1} = -\gamma_3 \sum_{i=1}^{Nn} (z_i^2(t))^{\frac{\mu+1}{2}} \\ &\le -\gamma_3(Nn)^{\frac{1-\mu}{2}}\Big(\sum_{i=1}^{Nn} z_i^2(t)\Big)^{\frac{\mu+1}{2}} = -\gamma_3(Nn)^{\frac{1-\mu}{2}}\Big(\delta^T(t)(\mathcal{L}^{\sigma(t)}\mathcal{L}^{\sigma(t)} \otimes PP)\delta(t)\Big)^{\frac{\mu+1}{2}}.\end{aligned} \tag{29}$$

Substituting (27), (28) and (29) into (25) yields that

$$\begin{aligned}\dot{V}(t) \le&\delta^T(t)\Big(\mathcal{L}^{\sigma(t)} \otimes \frac{PA + A^T P}{2} - (\gamma_1 - 2\eta_{\max})(\mathcal{L}^{\sigma(t)}\mathcal{L}^{\sigma(t)} \otimes PBB^T P)\Big)\delta(t)- \\ &\alpha\Big(\sum_{i=1}^{N} \phi_i(t)\Big)^{\frac{1}{2}} - \gamma_2\Big(\delta^T(t)(\mathcal{L}^{\sigma(t)}\mathcal{L}^{\sigma(t)} \otimes PP)\delta(t)\Big)^{\frac{1}{2}}- \\ &\gamma_3(Nn)^{\frac{1-\mu}{2}}\Big(\delta^T(t)(\mathcal{L}^{\sigma(t)}\mathcal{L}^{\sigma(t)} \otimes PP)\delta(t)\Big)^{\frac{\mu+1}{2}} - \beta N^{\frac{1-\mu}{2}}\Big(\sum_{i=1}^{N} \phi_i(t)\Big)^{\frac{\mu+1}{2}} + N\tau.\end{aligned} \tag{30}$$

According to Lemma 2, there exists a unitary matrix $U \in R^{N\times N}$ such that $U^T\mathcal{L}^{\sigma(t)}U = \Lambda^{\sigma(t)} = diag\{\lambda_1(\mathcal{L}^{\sigma(t)}), \lambda_2(\mathcal{L}^{\sigma(t)}), \cdots, \lambda_N(\mathcal{L}^{\sigma(t)})\}$ holds. Define $\bar{\delta}(t) = col(\bar{\delta}_1(t), \bar{\delta}_2(t), \cdots, \bar{\delta}_N(t)) = (U^T \otimes I_n)\delta(t)$, then (30) can be written as

$$\begin{aligned}
\dot{V}(t) \leq & \bar{\delta}^T(t)\Big(\Lambda^{\sigma(t)} \otimes \frac{PA + A^T P}{2} - (\gamma_1 - 2\eta_{\max})(\Lambda^{\sigma(t)}\Lambda^{\sigma(t)} \otimes PBB^T P)\Big)\bar{\delta}(t) - \\
& \alpha\Big(\sum_{i=1}^{N}\phi_i(t)\Big)^{\frac{1}{2}} - \gamma_2\Big(\bar{\delta}^T(t)(\Lambda^{\sigma(t)}\Lambda^{\sigma(t)} \otimes PP)\bar{\delta}(t)\Big)^{\frac{1}{2}} - \\
& \gamma_3(Nn)^{\frac{1-\mu}{2}}\Big(\bar{\delta}^T(t)(\Lambda^{\sigma(t)}\Lambda^{\sigma(t)} \otimes PP)\bar{\delta}(t)\Big)^{\frac{\mu+1}{2}} - \beta N^{\frac{1-\mu}{2}}\Big(\sum_{i=1}^{N}\phi_i(t)\Big)^{\frac{\mu+1}{2}} + N\tau \\
\leq & \sum_{i=1}^{N}\lambda_i(\mathcal{L}^{\sigma(t)})\bar{\delta}_i^T(t)\Big(\frac{PA + A^T P}{2} - (\gamma_1 - 2\eta_{\max})\lambda_i(\mathcal{L}^{\sigma(t)})PBB^T P\Big)\bar{\delta}_i(t) - \\
& \alpha\Big(\sum_{i=1}^{N}\phi_i(t)\Big)^{\frac{1}{2}} - \gamma_2\Big(\sum_{i=1}^{N}\lambda_i^2(\mathcal{L}^{\sigma(t)})\bar{\delta}_i^T(t)PP\bar{\delta}_i(t)\Big)^{\frac{1}{2}} - \\
& \gamma_3(Nn)^{\frac{1-\mu}{2}}\Big(\sum_{i=1}^{N}\lambda_i^2(\mathcal{L}^{\sigma(t)})\bar{\delta}_i^T(t)PP\bar{\delta}_i(t)\Big)^{\frac{\mu+1}{2}} - \beta N^{\frac{1-\mu}{2}}\Big(\sum_{i=1}^{N}\phi_i(t)\Big)^{\frac{\mu+1}{2}} + N\tau.
\end{aligned} \tag{31}$$

According to the conditions of Theorem 1, (31) can be written as

$$\begin{aligned}
\dot{V}(t) \leq & -\frac{1}{2}\gamma\sum_{i=1}^{N}\lambda_i(\mathcal{L}^{\sigma(t)})\bar{\delta}_i^T(t)P\bar{\delta}_i(t) - \alpha\Big(\sum_{i=1}^{N}\phi_i(t)\Big)^{\frac{1}{2}} - \beta N^{\frac{1-\mu}{2}}\Big(\sum_{i=1}^{N}\phi_i(t)\Big)^{\frac{\mu+1}{2}} - \\
& \alpha\Big(\frac{1}{2}\sum_{i=1}^{N}\lambda_i(\mathcal{L}^{\sigma(t)})\bar{\delta}_i^T(t)P\bar{\delta}_i(t)\Big)^{\frac{1}{2}} - \beta N^{\frac{1-\mu}{2}}\Big(\frac{1}{2}\sum_{i=1}^{N}\lambda_i(\mathcal{L}^{\sigma(t)})\bar{\delta}_i^T(t)P\bar{\delta}_i(t)\Big)^{\frac{\mu+1}{2}} + N\tau \\
\leq & -\gamma V_1(t) - \alpha\Big(V_1^{\frac{1}{2}}(t) + V_2^{\frac{1}{2}}(t)\Big) - \beta N^{\frac{1-\mu}{2}}\Big(V_1^{\frac{\mu+1}{2}}(t) + V_2^{\frac{\mu+1}{2}}(t)\Big) + N\tau \\
\leq & -\alpha V^{\frac{1}{2}}(t) - \beta(2N)^{\frac{1-\mu}{2}}V^{\frac{\mu+1}{2}}(t) + N\tau.
\end{aligned} \tag{32}$$

Then, according to Lemma 1, it can be known from (32) that the system (16) is globally practical fixed-time stable. The upper bound of the settling time is $T \leq T_{\max} = \frac{2}{\alpha\kappa} + \frac{2^{\frac{\mu+1}{2}}N^{\frac{\mu-1}{2}}}{\beta\kappa(\mu-1)}$, and the residual error set of the solution is $\{\lim_{t\to T}\delta(t)|V(t) \leq \min\{(\frac{N\tau}{\alpha(1-\kappa)})^2, (2N)^{\frac{\mu-1}{\mu+1}}(\frac{N\tau}{\beta(1-\kappa)})^{\frac{2}{\mu+1}}\}\}$. Thus, the considered multi-agent system (3) achieves the practical fixed-time average consensus. The proof of Theorem 1 is completed. □

Theorem 2. For the multi-agent system (3) satisfying Assumptions 1-3, the individual controller and event-triggered condition are given by (4) and (9), respectively, then the Zeno behavior will not occur in the system (3).

Proof. By proving that the minimum event interval $t_{k+1}^i - t_k^i$ is strictly positive, it is proved that the system will not have Zeno behavior under the proposed dynamic event-triggered mechanism. Since the adjacent topology switching instant satisfies $\tau < t_{s+1} - t_s < \mathcal{T}$ and τ is a positive constant, it can be seen that topology switching will certainly not lead to Zeno behavior. In general, the exclusion of Zeno behavior under switching topologies needs to consider the following three cases.

1. Consider that the time between adjacent events satisfies $t_{k+1}^i - t_k^i < \tau$, and there is no topology switching in the triggering interval $[t_k^i, t_{k+1}^i)$. At this time, the communication topology is fixed. According to (6), we can get

$$\begin{aligned}
e_i(t) =& \gamma_1 B^T P\xi_i(t_k^i) + \gamma_2 K sign(P\xi_i(t_k^i)) + \gamma_3 K sig^{\mu}(P\xi_i(t_k^i)) \\
& - \gamma_1 B^T P\xi_i(t) - \gamma_2 K sign(P\xi_i(t)) - \gamma_3 K sig^{\mu}(P\xi_i(t)) \\
=& \gamma_1 B^T P e_{1,i}(t) + \gamma_2 K e_{2,i}(t) + \gamma_3 K e_{3,i}(t),
\end{aligned} \tag{33}$$

where $e_{1,i}(t) = \xi_i(t_k^i) - \xi_i(t)$, $e_{2,i}(t) = sign(P\xi_i(t_k^i)) - sign(P\xi_i(t))$, $e_{3,i}(t) = sig^{\mu}(P\xi_i(t_k^i)) - sig^{\mu}(P\xi_i(t))$.

For $t \in [t_k^i, t_{k+1}^i)$, there is

$$\begin{aligned}
\frac{d\|e_i(t)\|}{dt} &\leq \|\dot{e}_i(t)\| = \|\gamma_1 B^T P\dot{e}_{1,i}(t) + \gamma_2 K\dot{e}_{2,i}(t) + \gamma_3 K\dot{e}_{3,i}(t)\| \\
&\leq \|\gamma_1 B^T P\|\|\dot{e}_{1,i}(t)\| + \|\gamma_2 K\|\|\dot{e}_{2,i}(t)\| + \|\gamma_3 K\|\|\dot{e}_{3,i}(t)\|.
\end{aligned} \tag{34}$$

Calculate $\|\dot{e}_{1,i}(t)\|$, $\|\dot{e}_{2,i}(t)\|$ and $\|\dot{e}_{3,i}(t)\|$ respectively as follows:

$$\begin{aligned}\|\dot{e}_{1,i}(t)\| \leq \|\dot{\xi}_i(t)\| &= \|A\xi_i(t) + B\sum_{j=1}^{N} a_{ij}(u_i(t)-u_j(t))\| \\ &\leq \|A\|\|\xi_i(t)\| + \|B\|\|\sum_{j=1}^{N} l_{ij}u_j(t)\|,\end{aligned} \tag{35}$$

$$\begin{aligned}\|\dot{e}_{2,i}(t)\| &\leq \|\frac{d}{dt}sign(P\xi_i(t))\| \approx \|\frac{d}{dt}tanh(\varepsilon P\xi_i(t))\| \\ &\leq \varepsilon\|1-tanh^2(\varepsilon P\xi_i(t))\|\|P\|\|\dot{\xi}_i(t)\| \\ &\leq \varepsilon\|P\|\Big(\|A\|\|\xi_i(t)\| + \|B\|\|\sum_{j=1}^{N} l_{ij}u_j(t)\|\Big),\end{aligned} \tag{36}$$

$$\begin{aligned}\|\dot{e}_{3,i}(t)\| &\leq \|\frac{d}{dt}sig^{\mu}(P\xi_i(t))\| \leq \mu\|P\|\|P\xi_i(t)\|^{\mu-1}\|\dot{\xi}_i(t)\| \\ &\leq \mu(\xi^T(t)(I_N\otimes PP)\xi(t))^{\frac{\mu-1}{2}}\|P\|\|\dot{\xi}_i(t)\| \\ &\leq \mu(2\lambda_N(\mathcal{L})\lambda_{\max}(P)V_1(t))^{\frac{\mu-1}{2}}\|P\|\|\dot{\xi}_i(t)\| \\ &\leq \mu(2\lambda_N(\mathcal{L})\lambda_{\max}(P)V_1(0))^{\frac{\mu-1}{2}}\|P\|\Big(\|A\|\|\xi_i(t)\| + \|B\|\|\sum_{j=1}^{N} l_{ij}u_j(t)\|\Big).\end{aligned} \tag{37}$$

In addition, there is

$$\begin{aligned}\|\xi_i(t)\| &\leq \|P^{-1}\|\|P\xi_i(t)\| \leq \|P^{-1}\|(\xi^T(t)(I_N\otimes PP)\xi(t))^{\frac{1}{2}} \\ &\leq \|P^{-1}\|(2\lambda_N(\mathcal{L})\lambda_{\max}(P)V_1(t))^{\frac{1}{2}} \leq \|P^{-1}\|(2\lambda_N(\mathcal{L})\lambda_{\max}(P)V_1(0))^{\frac{1}{2}}.\end{aligned} \tag{38}$$

Substituting (35), (36), (37) and (38) into (34) yields that

$$\begin{aligned}\frac{d\|e_i(t)\|}{dt} \leq& \|\gamma_1 B^TP\|\Big(\Pi^{\frac{1}{2}}\|A\|\|P^{-1}\| + \|B\|\|\sum_{j=1}^{N} l_{ij}u_j(t)\|\Big) + \varepsilon\|\gamma_2K\|\|P\|\Big(\Pi^{\frac{1}{2}}\|A\|\|P^{-1}\| + \|B\|\|\sum_{j=1}^{N} l_{ij}u_j(t)\|\Big) \\ &+ \mu\Pi^{\frac{\mu-1}{2}}\|\gamma_3K\|\|P\|\Big(\Pi^{\frac{1}{2}}\|A\|\|P^{-1}\| + \|B\|\|\sum_{j=1}^{N} l_{ij}u_j(t)\|\Big) \\ \leq& \gamma_1\Pi^{\frac{1}{2}}\|B^T\|\|P\|\|A\|\|P^{-1}\| + \gamma_2\varepsilon\Pi^{\frac{1}{2}}\|K\|\|P\|\|A\|\|P^{-1}\| + \gamma_3\mu\Pi^{\frac{\mu}{2}}\|K\|\|P\|\|A\|\|P^{-1}\| \\ &+ \gamma_1\|B^T\|\|P\|\|B\|\|\sum_{j=1}^{N} l_{ij}u_j(t)\| + \gamma_2\varepsilon\|K\|\|P\|\|B\|\|\sum_{j=1}^{N} l_{ij}u_j(t)\| + \gamma_3\mu\Pi^{\frac{\mu-1}{2}}\|K\|\|P\|\|B\|\|\sum_{j=1}^{N} l_{ij}u_j(t)\| \\ \leq& \Big(\gamma_1\Pi^{\frac{1}{2}}\|B^T\| + \gamma_2\varepsilon\Pi^{\frac{1}{2}}\|K\| + \gamma_3\mu\Pi^{\frac{\mu}{2}}\|K\|\Big)\|P\|\|A\|\|P^{-1}\| \\ &+ \Big(\gamma_1\|B^T\| + \gamma_2\varepsilon\|K\| + \gamma_3\mu\Pi^{\frac{\mu-1}{2}}\|K\|\Big)\|P\|\|B\|\|\sum_{j=1}^{N} l_{ij}u_j(t_{k'}^j)\| \\ =& \Delta_0 + \Delta_1(t_{k'}^j),\end{aligned} \tag{39}$$

where $t_{k'}^j$ is the most recent triggering instant for agent j, $\Pi = 2\lambda_N(\mathcal{L})\lambda_{\max}(P)V_1(0)$, $\Delta_0 = (\gamma_1\Pi^{\frac{1}{2}}\|B^T\| + \gamma_2\varepsilon\Pi^{\frac{1}{2}}\|K\| + \gamma_3\mu\Pi^{\frac{\mu}{2}}\|K\|)\|P\|\|A\|\|P^{-1}\|$, $\Delta_1(t_{k'}^j) = (\gamma_1\|B^T\| + \gamma_2\varepsilon\|K\| + \gamma_3\mu\Pi^{\frac{\mu-1}{2}}\|K\|)\|P\|\|B\|\|\sum_{j=1}^{N} l_{ij}u_j(t_{k'}^j)\|$.

Since $e_i(t_k^i) = 0$, there is

$$\|e_i(t)\| \leq \int_{t_k^i}^{t} \|\dot{e}_i(\zeta)\|d\zeta \leq \int_{t_k^i}^{t} (\Delta_0 + \Delta_1(t_{k'}^j))d\zeta. \tag{40}$$

According to the event-triggered condition (9), when $g_i(t^i_{k+1}) \geq f(\phi_i(t^i_{k+1}))$, the $k+1$-th event of agent i is triggered. Then there is

$$\|e_i(t^i_{k+1})\|^2 \geq \eta_i^2 \xi_i^T(t^i_{k+1}) PBB^T P\xi_i(t^i_{k+1}) + \eta_i \alpha \phi_i^{\frac{1}{2}}(t^i_{k+1}) + \eta_i \beta \phi_i^{\frac{\mu+1}{2}}(t^i_{k+1}). \tag{41}$$

According to (40) and (41), we can get

$$0 < \Theta \leq \|e_i(t^i_{k+1})\| \leq \int_{t^i_k}^{t^i_{k+1}} (\Delta_0 + \Delta_1(t^j_{k'}))d\zeta \leq (\Delta_0 + \Delta_1)(t^i_{k+1} - t^i_k), \tag{42}$$

where $\Delta_1 = \max\{\Delta_1(t^j_0), \Delta_1(t^j_1), \Delta_1(t^j_2), \cdots\}$, $\Theta = (\eta_i^2 \xi_i^T(t^i_{k+1}) PBB^T P\xi_i(t^i_{k+1}) + \eta_i \alpha \phi_i^{\frac{1}{2}}(t^i_{k+1}) + \eta_i \beta \phi_i^{\frac{\mu+1}{2}}(t^i_{k+1}))^{\frac{1}{2}}$. Further, it can be obtained that

$$t^i_{k+1} - t^i_k \geq \frac{\Theta}{\Delta_0 + \Delta_1} > 0. \tag{43}$$

Therefore, in case 1, Zeno behavior does not occur in the multi-agent system.

2. Consider that there is a certain topology switching in the triggering interval $[t^i_k, t^i_{k+1})$. At this time, it is best to assume that the switching instant is t_s, and the time interval $[t_s, t^i_{k+1})$ satisfies $t^i_{k+1} - t_s < \tau$. According to (5), t_s is regarded as a new triggering instant, then the exclusion of Zeno behavior of the system is the same as the first case.

3. Consider that the time interval $[t^i_k, t_s)$ satisfies $t_s - t^i_k < \tau$. Since t^i_{k+1} is earlier than t_{s+1}, the next triggering interval is $[t_s, t^i_{k+1})$. Therefore, it is only necessary to prove that the minimum time interval $t^i_{k+1} - t_s$ is strictly positive. At this time, the exclusion of Zeno behavior of the system is the same as the second case.

It is worth mentioning that for the case of multiple topology switching in the triggering interval $[t^i_k, t^i_{k+1})$, the problem can be transformed into the case of fixed topology by segmenting the time interval, which can easily exclude the Zeno behavior of the system. In summary, Zeno behavior does not occur in the system (3). □

Remark 3. A sufficient and necessary condition for the existence of the solution $P > 0$ of LMI (17) is that the matrix pair (A, B) is stable.[33] Assumption 1 imposes a restriction on the control matrix B with row full rank, which indicates that the matrix pair (A, B) is controllable. This is a sufficient condition for the solvability of the LMI (17).

Remark 4. According to the definition of $V(t)$, $2V(t) \geq \delta^T(t)(\mathcal{L}^{\sigma(t)} \otimes P)\delta(t)$ holds, so there is $\delta^T(t)\delta(t) \leq \frac{2V(t)}{\lambda_{2-\min}\lambda_{\min}(P)}$, and further there is $\|\delta_i(t)\| \leq (\frac{2V(t)}{\lambda_{2-\min}\lambda_{\min}(P)})^{\frac{1}{2}}$. Then, according to the settling time and the residual error set of the solution of the system (16), it can be seen that when $t \to T$ and $t \geq T$, $\|x_i(t) - \frac{1}{N}\sum_{j=1}^N x_j(t)\| \leq (\frac{2\min\{(\frac{N\tau}{\alpha(1-\kappa)})^2, (2N)^{\frac{\mu-1}{\mu+1}}(\frac{N\tau}{\beta(1-\kappa)})^{\frac{2}{\mu+1}}\}}{\lambda_{2-\min}\lambda_{\min}(P)})^{\frac{1}{2}}$ holds. Thus, the multi-agent system (3) achieves the practical fixed-time average consensus.

Remark 5. The derivative of the sign function with respect to time t is required in (36), but the sign function is not continuous, we consider using the continuous hyperbolic tangent function to approximate the sign function,[22,34] namely $sign(P\xi_i(t)) \approx tanh(\varepsilon P\xi_i(t))$, where $\varepsilon \gg 1$.

4 | NUMERICAL SIMULATION

In this section, the effectiveness of the theoretical results is verified by an example simulation. Then the influence of the relevant parameters in the designed dynamic event-triggered control protocol on the number of triggering events of the system is analyzed. Finally, a comparative experiment shows that the dynamic event-triggered mechanism based on auxiliary dynamic variables can avoid a large number of triggering events and save more resources.

4.1 | Theoretical verification

Consider that the multi-agent system consists of four agents with general continuous linear dynamic model, namely $N = 4$. The switching communication topologies and switching mechanism between agents are shown in Figure 1.

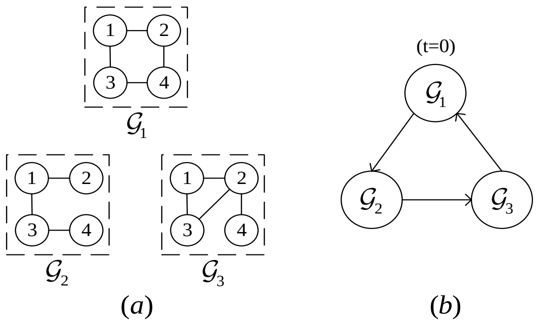

Figure 1 (a):Communication topologies. (b):State switching mechanism.

Suppose that the switching topologies $\mathcal{G}^{\sigma(t)}$ are determined by the following switching signal.

$$\sigma(t) = \begin{cases} 1, & \Omega T_s \le t < (\Omega + \frac{1}{3})T_s \\ 2, & (\Omega + \frac{1}{3})T_s \le t < (\Omega + \frac{2}{3})T_s \\ 3, & (\Omega + \frac{2}{3})T_s \le t < (\Omega + 1)T_s \end{cases}$$

where $T_s = 0.15$s is the switching period and Ω is a non-negative integer. It can be seen that Assumption 2 and Assumption 3 are satisfied. In order to facilitate the calculation and analysis, the weight of each connection edge in the system communication topology is taken as 1. According to Figure 1, $\lambda_{2-\min} = 0.5858$ can be obtained.

For the dynamic model of multi-agent system (3), the system matrices are selected as

$$A = \begin{bmatrix} -1 & 4 \\ -5 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

It can be seen that Assumption 1 holds. Based on the Assumption 1, the matrix K can be obtained as follows:

$$K = \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Take $\gamma = 2$ and solve LMI (17), the matrix P can be obtained as follows:

$$P = \begin{bmatrix} 1.0370 & -0.0574 \\ -0.0574 & 0.8901 \end{bmatrix}.$$

The initial states of the agents are taken as $x_1(0) = [32, 12]^T$, $x_2(0) = [-8, -4]^T$, $x_3(0) = [16, -10]^T$, and $x_4(0) = [-24, 8]^T$. For the controller (4), the event-triggered function (7) and the auxiliary dynamic variable (8), the design parameters are selected as $\mu = 7/5, \gamma_1 = 3.64, \gamma_2 = 1.74, \gamma_3 = 1.68, \eta_i = 0.9, \alpha = 1.48, \beta = 0.5, \tau = 0.001, \phi_i(0) = 162$ and $\kappa = 0.8$, respectively. It can be verified that the design parameters satisfy the conditions of Theorem 1. At this time, the simulation results are shown in Figure 2-Figure 5.

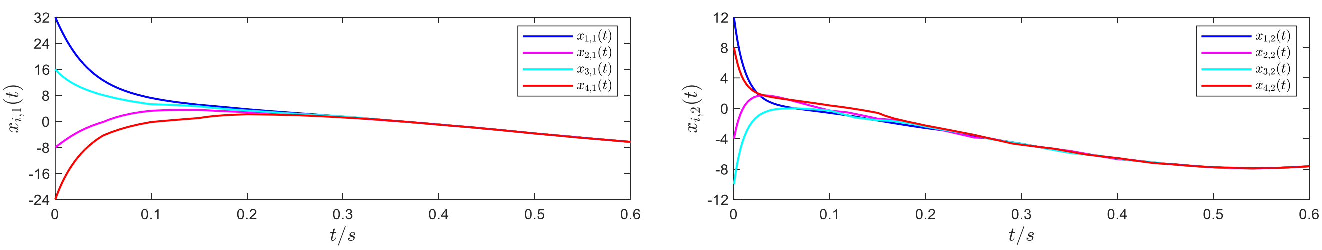

Figure 2 State evolution of each agent.

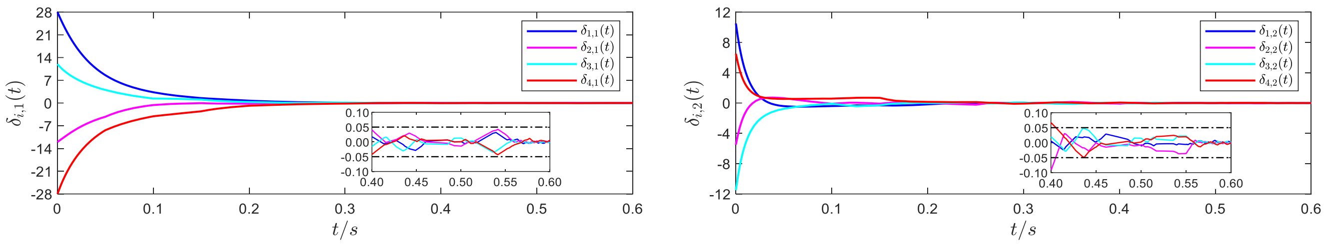

Figure 3 Average consensus error evolution of each agent.

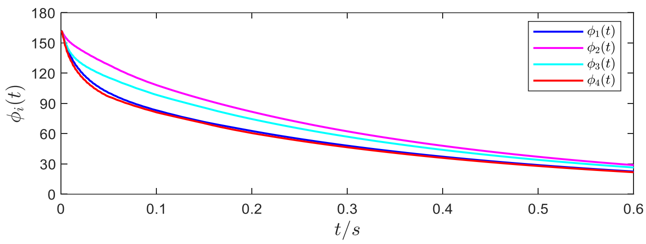

Figure 4 Trajectory of auxiliary dynamic variable of each agent.

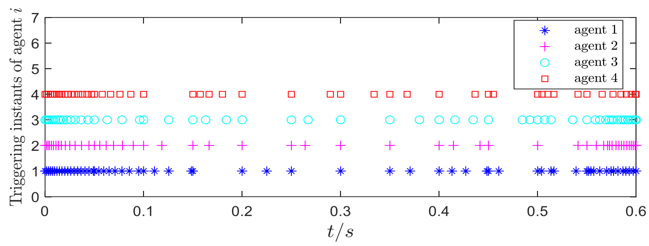

Figure 5 Triggering instants of each agent.

Figure 2 shows the state evolution of each agent, and Figure 3 shows the average consensus error evolution of each agent. It can be seen that the states of the four agents achieve consensus at $t = 0.4$s, and the average consensus error of each agent converges within ± 0.005. In addition, it can be found that the system can achieve the practical fixed-time consensus under different initial states, and Theorem 1 is verified. Figure 4 shows the trajectories of the auxiliary dynamic variables. It can be seen that the auxiliary dynamic variables converge to the bounded region of the 0 neighborhood when the system achieves consensus, and $\phi_i(t) > 0$ is verified. Figure 5 shows the respective triggering instants of the four agents. It can be seen that there is no Zeno behavior in the system, and Theorem 2 is verified. The results show that the control protocol based on dynamic event-triggered mechanism for the practical fixed-time average consensus of general linear multi-agent systems is effective.

4.2 | Parametric analysis

Different control parameters produce different control effects. Designing reasonable parameters can reduce the number of triggering events while ensuring that the system has good convergence performance. Based on the simulation example in Section 4.1, this section analyzes the influence of the design parameters $\gamma_1, \gamma_2, \gamma_3, \mu, \eta_i, \alpha, \beta, \tau, \phi_i(0)$ on the number of triggering events of the system. When other parameters are fixed, the relationships between the parameters studied and the total number of triggering events of the multi-agent system are shown in Figure 6-Figure 14.

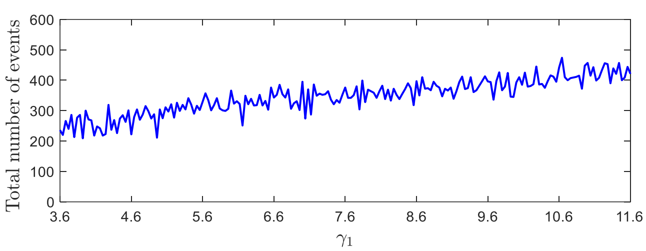

Figure 6 The total number of events varies with γ_1.

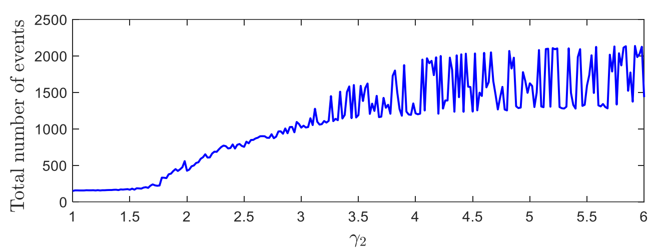

Figure 7 The total number of events varies with γ_2.

Figure 6-Figure 9 shows the influence of controller parameters on the number of triggering events. It can be seen that with the increase of γ_1, γ_2 and γ_3, the total number of events shows a trend of local fluctuation and overall increase. With the increase

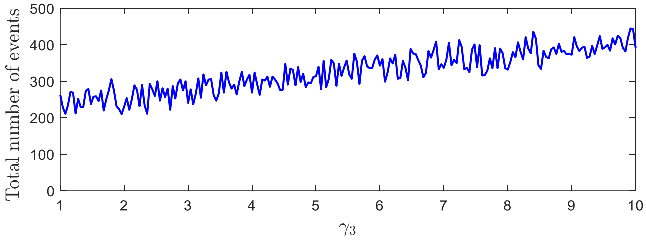

Figure 8 The total number of events varies with γ_3.

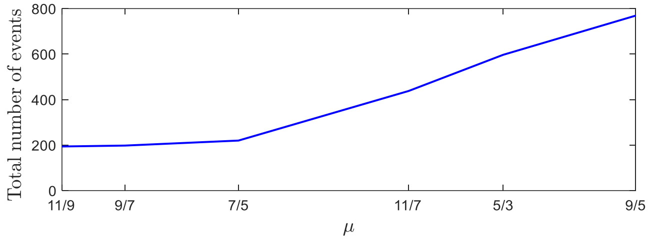

Figure 9 The total number of events varies with μ.

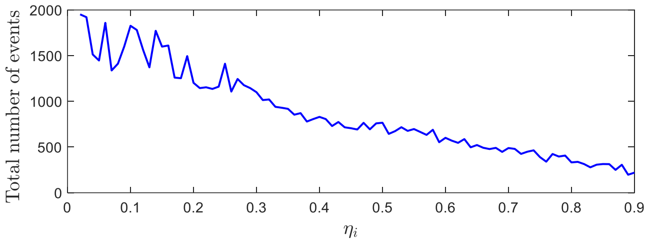

Figure 10 The total number of events varies with η_i.

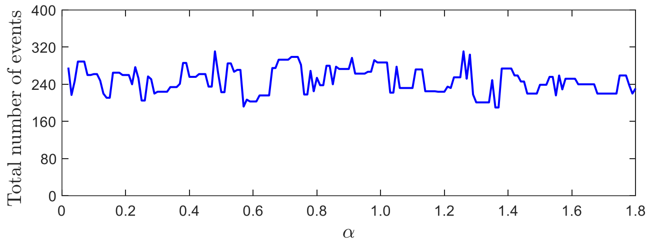

Figure 11 The total number of events varies with α.

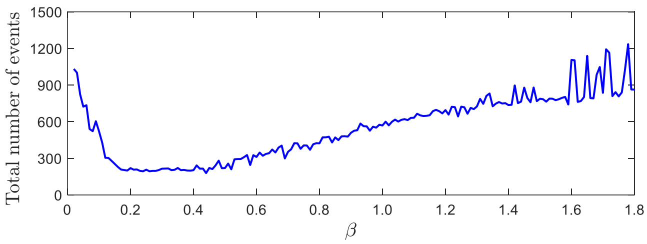

Figure 12 The total number of events varies with β.

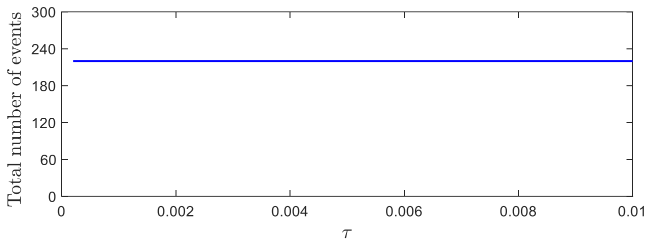

Figure 13 The total number of events varies with τ.

of μ, the total number of events shows a monotonically increasing trend. Figure 10-Figure 14 shows the influence of triggering condition parameters on the number of triggering events. It can be seen that within a certain range of meeting the system requirements, the total number of events decreases oscillatingly with the increase of η_i, fluctuates in a small range with the increase of α, decreases oscillatingly first and then increases oscillatingly with the increase of β, remains unchanged with the increase of τ, and decreases oscillatingly first and then fluctuates steadily with the increase of $\phi_i(0)$.

The method of adjusting parameters is as follows. 1. The triggering condition parameters are initially selected according to the design requirements. That is, the values of $\eta_i, \alpha, \beta, \tau$ and $\phi_i(0)$ are pre-determined. 2. The values of the controller parameters γ_1, γ_2 and γ_3 are determined by the sufficient conditions of Theorem 1, and the value of μ is preliminarily selected. 3. According to the results of parametric analysis, adjust the values of $\eta_i, \alpha, \beta, \tau, \phi_i(0)$. 4. Correct the values of $\gamma_1, \gamma_2, \gamma_3$ and μ again. In

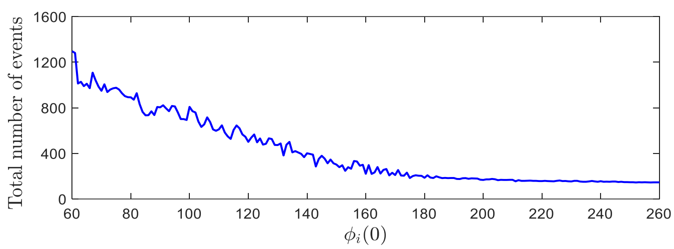

Figure 14 The total number of events varies with $\phi_i(0)$.

addition, when adjusting the parameters, it is found that the number of triggering events is inversely related to the convergence speed. The parameter τ is only related to the convergence error, but has little effect on the number of triggering events and the convergence speed. Therefore, in practical engineering applications, designers can balance the number of triggering events and the convergence speed in the process of parameter adjustment according to the needs of the system, and reduce the convergence error by adjusting the parameter τ.

4.3 | Comparative experiment

In order to present the advantages of the control protocol designed in this paper in reducing the number of system triggering events, the results of the static event-triggered control protocol are given below for comparison.

The static event-triggered condition is given as

$$t_{k+1}^i = \inf\{t > t_k^i | g_i(t) \geq 0\}, k = 0, 1, \cdots . \tag{44}$$

When the parameters are the same as Section 4.1, for the multi-agent system (3) with controller (4) and event-triggered condition (44), the simulation results for the practical fixed-time average consensus are shown in Figure 15-Figure 17.

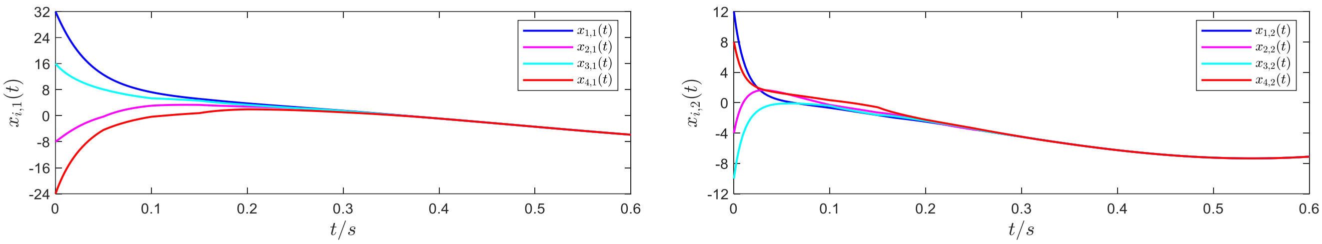

Figure 15 State evolution of each agent under the static event-triggered control.

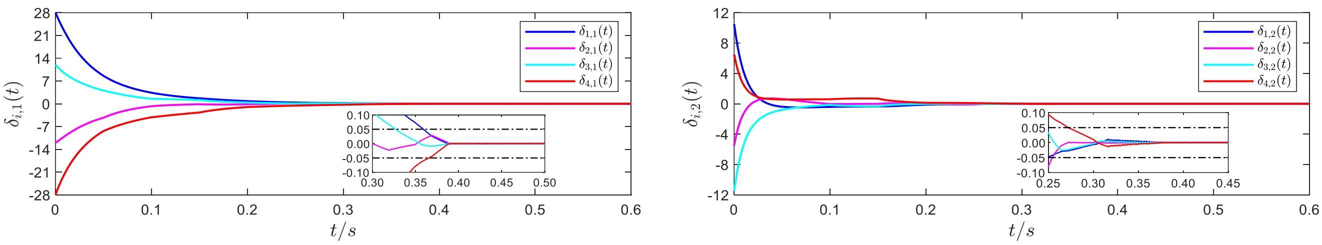

Figure 16 Average consensus error evolution of each agent under the static event-triggered control.

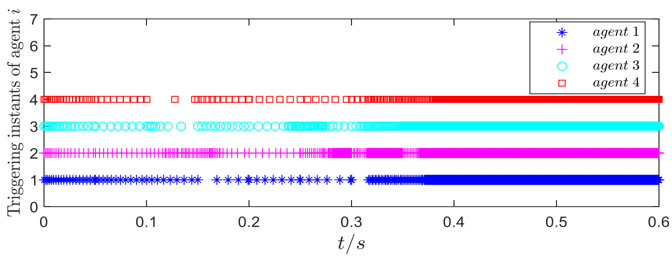

Figure 17 Triggering instants of each agent under the static event-triggered control.

Figure 15 and Figure 16 show the state evolution and the average consensus error evolution of each agent under the static event-triggered mechanism, respectively. Figure 17 shows the triggering instants of each agent under the static event-triggered mechanism. Compared with the simulation results in Section 4.1, it is found that the multi-agent system based on the dynamic event-triggered mechanism has fewer event-triggered moments at almost the same convergence rate.

Further, under two different event-triggered mechanisms, the number of triggering events of each agent in the first 0.6s is shown in Table 1.

Table 1 The number of triggering events of each agent under two event-triggered mechanisms.

Event-triggered mechanism	**The number of triggering events of each agent**			
	agent 1	**agent 2**	**agent 3**	**agent 4**
(9)	62	51	56	51
(44)	553	701	611	537

The results show that the dynamic event-triggered mechanism designed in this paper can avoid a large number of triggering events and significantly reduce the energy consumption of the multi-agent system and the update frequency of the controller compared to the static event-triggered mechanism.

5 | CONCLUSION

In this paper, the practical fixed-time average consensus problem of general continuous linear multi-agent systems under switching topologies is studied, and a distributed control protocol based on dynamic event-triggered mechanism is proposed, which significantly reduces the number of triggering events and can effectively reduce the energy dissipation of the system and the update frequency of the controller. Firstly, a fixed-time consensus controller is designed for individual agent by using local information exchange between neighbors. Secondly, a dynamic event-triggered condition based on auxiliary dynamic variable is designed for individual agent to determine the triggering instants of each agent. Thirdly, the sufficient conditions for the multi-agent system to solve the practical fixed-time average consensus problem are given. Then, it is proved that the proposed protocol can make the multi-agent system achieve the practical fixed-time average consensus without Zeno behavior by using Lyapunov stability theory, linear matrix inequality and algebraic graph theory. Finally, numerical simulation is carried out. The results show that the control protocol designed in this paper and based on dynamic event-triggered mechanism for the practical fixed-time average consensus of general linear multi-agent systems is effective and feasible, and compared with the static event-triggered mechanism, the dynamic event-triggered mechanism can avoid a lot of triggering events. Future work will study the fixed-time leader-follower consensus of general linear multi-agent systems based on the dynamic event-triggered mechanism.

ACKNOWLEDGMENTS

The authors would like to thank the editor and all anonymous reviewers for their valuable comments and suggestions. This work was supported by the National Natural Science Foundation of China (61663020), the Gansu Provincial University Industry Support Plan Project (2022CYZC-33), and the Open Subject of State Key Laboratory of Structural Analysis for Industrial Equipment of Dalian University of Technology (ZG22119).

CONFLICT OF INTEREST

The authors declare no potential conflict of interests.

References

1. Wang Y, Cheng Z, Xiao M. UAVs'formation keeping control based on multi–agent system consensus. *IEEE Access* 2020; 8: 49000–49012.

2. Ge X, Han Q. Distributed formation control of networked multi-agent systems using a dynamic event-triggered communication mechanism. *IEEE T Ind Electron* 2017; 64(10): 8118–8127.

3. Ren W, Atkins E. Distributed multi-vehicle coordinated control via local information exchange. *Int J Robust Nonlin* 2007; 17(10-11): 1002–1033.

4. Ren W. Formation keeping and attitude alignment for multiple spacecraft through local interactions. *J Guid Control Dynam* 2007; 30(2): 633–638.

5. Wang H, Gao J, Su ea. Fixed-time formation of AUVs based on event-triggered control. *IEEE/CAA JAS* 2022; 48(9): 2277–2287.

6. Schiffer J, Seel T, Raisch J. Voltage stability and reactive power sharing in inverter-based microgrids with consensus-based distributed voltage control. *IEEE T Contr Syst T* 2016; 24(1): 96–109.

7. Fan Y, Hu G, Egerstedt M. Distributed reactive power sharing control for microgrids with event-triggered communication. *IEEE T Contr Syst T* 2017; 25(1): 118–128.

8. Hu S, Qiu J, Chen ea. Dynamic event-triggered control for leader-following consensus of multiagent systems with the estimator. *IET Control Theory A* 2022; 16(4): 475–484.

9. Polyakov A. Nonlinear feedback design for fixed-time stabilization of linear control systems. *IEEE T Automat Contr* 2012; 57(8): 2106–2110.

10. Hong H, Yu W, Wen G. Distributed robust fixed-time consensus for nonlinear and disturbed multiagent systems. *IEEE T Syst Man Cy-S* 2017; 47(7): 1464–1473.

11. Defoort M, Polyakov A, Demesure G. Leader-follower fixed-time consensus for multi-agent systems with unknown nonlinear inherent dynamics. *IET Control Theory A* 2015; 9(14): 2165–2170.

12. Wang Q, Wang Y, Sun C. Fixed-time consensus of multi-agent systems with directed and intermittent communications. *Asian J Control* 2017; 19(1): 95–105.

13. Zhang B, Jia Y. Fixed-time consensus protocols for multiagent systems with linear and nonlinear state measurements. *Nonlinear Dynam* 2015; 82(4): 1683–1690.

14. Ning B, Jin J, Zheng J. Fixed-time consensus for multi-agent systems with discontinuous inherent dynamics over switching topology. *INT J Syst Sci* 2017; 48(10): 2023–2032.

15. Zuo Z, Tian B, Defoort M. Fixed-time consensus tracking for multiagent systems with high-order integrator dynamics. *IEEE T Automat Contr* 2018; 63(2): 563–570.

16. Wang H, Yu W, Wen ea. Fixed-time consensus tracking of multi-agent systems under a directed communication topology. In: Paper presented at: 12th International Conference on Control and Automation (ICCA). IEEE. ; 2016; Kathmandu, Nepal.

17. Liu J, Yu Y, Sun ea. Distributed event-triggered fixed-time consensus for leader-follower multiagent systems with nonlinear dynamics and uncertain disturbances. *Int J Robust Nonlin* 2018; 28(11): 3543–3559.

18. Liu J, Zhang Y, Yu ea. Fixed-time event-triggered consensus for nonlinear multiagent systems without continuous communications. *IEEE T Syst Man Cy-S* 2019; 49(11): 2221–2229.

19. Liu J, Zhang Y, Sun ea. Fixed-time consensus of multi-agent systems with input delay and uncertain disturbances via event-triggered control. *Inform Sciences* 2019; 480: 261–272.

20. Zhou D, Zhang A, Yang P. Fixed-time event-triggered consensus of second-order multi-agent systems with fully continuous communication free. *IET Control Theory A* 2020; 14(16): 2385–2394.

21. Liu J, Zhang Y, Yu ea. Fixed-time leader-follower consensus of networked nonlinear systems via event/self-triggered control. *IEEE T Neur Net Lear* 2020; 31(11): 5029–5037.

22. Ai X, Wang L. Distributed fixed-time event-triggered consensus of linear multi-agent systems with input delay. *Int J Robust Nonlin* 2021; 31(7): 2526–2545.

23. Liu J, Yu Y, Xu ea. Fixed-time average consensus of nonlinear delayed MASs under switching topologies: an event-based triggering approach. *IEEE T Syst Man Cy-S* 2022; 52(5): 2721–2733.

24. Liu J, Ran G, Wu ea. Dynamic event-triggered practical fixed-time consensus for nonlinear multiagent systems. *IEEE T Circuits-II* 2022; 69(4): 2156–2160.

25. Liu J, Wu Y, Sun M. Fixed-time cooperative tracking for delayed disturbed multi-agent systems under dynamic event-triggered control. *IEEE/CAA JAS* 2022; 9(5): 930–933.

26. Sun M, Ren L, Liu J. Dynamic event-triggered fixed-time average consensus control of multi-agent systems under switching topologies. *IEEE/CAA JAS* 2022; 48(4): 1–11.

27. Zhu Z, Xia Y, Fu M. Attitude stabilization of rigid spacecraft with finite-time convergence. *Int J Robust Nonlin* 2011; 21(6): 686–702.

28. Ran G, Liu J, Li C. Event-based finite-time consensus control of second-order delayed multi-agent systems. *IEEE T Circuits-II* 2021; 68(1): 276–280.

29. Hong H, Yu W, Yu X. Fixed-time connectivity-preserving distributed average tracking for multiagent systems. *IEEE T Circuits-II* 2017; 64(10): 1192–1196.

30. Liu J, Yu Y, Wang Q. Fixed-time event-triggered consensus control for multi-agent systems with nonlinear uncertainties. *Neurocomputing* 2017; 260(18): 497–504.

31. Li S, Du H, Lin X. Finite-time consensus algorithm for multi-agent systems with double-integrator dynamics. *Automatica* 2011; 47(8): 1706–1712.

32. Yi X, Liu K, Dimarogonas D. Dynamic event-triggered and self-triggered control for multi-agent systems. *IEEE T Automat Contr* 2019; 64(8): 3300–3307.

33. Cheng B, Li Z. Fully distributed event-triggered protocols for linear multiagent networks. *IEEE T Automat Contr* 2019; 64(4): 1655–1662.

34. Nair R, Behera L, Kumar S. Event-triggered finite-time integral sliding mode controller for consensus-based formation of multirobot systems with disturbances. *IEEE T Contr Syst T* 2019; 27(1): 39–47.

How to cite this article: Williams K., B. Hoskins, R. Lee, G. Masato, and T. Woollings (2016), A regime analysis of Atlantic winter jet variability applied to evaluate HadGEM3-GC2, *Q.J.R. Meteorol. Soc., 2017;00:1–6*.